Abstract. This paper presents the study of a real-time procurement and production mechanism for a three-stage supply chain system with multiple suppliers, subject to unexpected disruptions. In the first part of the paper, a mathematical model was developed for the optimization of replenishment and production decisions for each node after the occurrence of a transportation disruption. In addition, an experiment was conducted to study the effects of disruptions to the system using predefined scenarios, where the supplier’s prioritization of disruption mitigation strategies was explored. Various disruption scenarios were predefined by combining different disruption types and locations as well as different combinations of suppliers. It will be shown that the solution to the transportation disruption was more sensitive to the lot size when the lost sales cost was large. However, when the lost sales cost was low, the sensitivity to the lot size decreased, and the setup cost and inventory holding cost became more sensitive. The benefits of optimization has led to an increased understanding of the impact of random disruption events on the behaviour of the total system, besides determining the priorities for risk mitigation efforts.

Keywords: Supply chain management, multiple suppliers, disruption, and recovery.

1 Introduction

There has been a growing awareness in recent years regarding the importance of supply chain risk management. For the purposes of business continuity, it is important for a firm to know and understand the different risk properties of each of the partners in its supply chain (SC). The literature on supply chain disruption includes studies that explored the optimization of inventory replenishment decisions in single and two-stage supply chain structures in the aftermath of a disruption [1],[2],[3],[4]. These studies individually focussed on a single type of disruption. However, the designs of modern supply chains are more complex in nature and consist of multiple stages with multiple entities in each stage. Furthermore, in reality, a supply chain node typically experiences a variety of disruptions [5], and their impact normally cascades through the other nodes of the supply chain structure [6].

Under the multi-echelon SC context, many studies have focused on exploring optimal strategies rather than on optimizing inventory replenishment decisions in times of disruptions [7],[8],[9]. In addition, several researches have studied models with multiple unreliable suppliers who supply the same parts, where supply uncertainty is taken into consideration in their models. Most of the works have been aimed at selecting which supplier to order from and in what quantity in order to hedge against disruptions, given different supplier assumptions [10],[11],[12],[13]. While the above works studied the problem of supplier selection or routine sourcing, none of them examined the effects of supply and transportation disruptions simultaneously in their models.

In a typical SC system, different suppliers exist with varying systems and operating costs. Moreover, each plant faces plant-specific, stationary demands. Consider a typical SC structure of a three-stage supply chain consisting of three supply nodes, namely a supplier node, a production node, and a customer or retailer node. In between each node are the transportation links. Disruptions may occur in the form of a transportation disruption (TDi) at one of the transportation links. A supply disruption (SDi) may also occur at one of the nodes as a consequence. Each supplier may have unequal optimal production lot sizes (Qi), depending on the different cost parameters of the plant. Therefore, when a disruption arises, it is obvious that the effects of the disruption on each plant will differ depending on these parameters.

This research investigates the recovery properties of different suppliers in a semi-integrated lot sizing problem for a three-stage supply chain system with multiple suppliers, subject to random disruption occurrences. The system in this study consisted of a partially integrated system of three different parts
suppliers, a manufacturer and a retailer. In addition to the optimization of the replenishment and manufacturing decisions of the intended system, one of the goals of this research was to obtain insights on priority selection for risk management strategies among the most critical suppliers. As it was assumed that the suppliers had been pre-selected and worked on a pre-determined supply portfolio, the study focused more on the decision of prioritizing suppliers for protection against disruptions by analysing the impact of different supplier disruptions on the total system recovery costs.

2 Model Development

2.1 Problem Description

Consider a manufacturer who has three suppliers, each supplier supplying different parts or raw material to the manufacturer, depending on the type of production system at hand. The manufacturer could be an assembly manufacturer, who assembles the parts into finished goods, or a pure manufacturer, who converts the raw material into the end product through manufacturing processes. To avoid confusion, the term parts will consistently be used throughout this paper. The finished goods are delivered to the retailer, who only has an inventory and does not carry out any production. Each supplier has a coordinated procurement system with the manufacturer. Under this policy, suppliers form a long–term partnership with the manufacturer and both parties share demand information. Meanwhile, information may or may not be shared between suppliers, or in other words, there exists a non-coordinated system between suppliers. Table 1 describes the specific disruption types that the system may experience and the designated location for each disruption.

<table>
<thead>
<tr>
<th>Disruption Type</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Transportation Disruption between Supplier 1 and Manufacturer</td>
</tr>
<tr>
<td>2</td>
<td>Transportation Disruption between Supplier 2 and Manufacturer</td>
</tr>
<tr>
<td>3</td>
<td>Transportation Disruption between Supplier 3 and Manufacturer</td>
</tr>
<tr>
<td>4</td>
<td>Transportation Disruption between Manufacturer and Retailer</td>
</tr>
</tbody>
</table>

Following a transportation disruption between Tiers 1 and 2, the retailer will experience a supply disruption of the same magnitude (Td) as a direct consequence. Although in real life the magnitude may be larger due to the “ripple effect”, which is the possible chain reaction of any event from one business unit to another, this study chose to ignore this feature so as to simplify the calculations. The assembly line at the manufacturer’s site will be interrupted due to shortages of parts from the disrupted supplier. Since there is no sharing of information between the suppliers, the non-disrupted suppliers will still deliver the current lot to the manufacturer. This will cause an inventory build-up of parts from the non-disrupted suppliers at the manufacturer’s site and create additional inventory holding costs for the manufacturer. It is only when the disruption is resolved that the plant can continue with its normal operations. In the case of a disruption, the non-disrupted suppliers will be requested to delay the next shipment of parts to the manufacturer until the parts inventory at the manufacturer’s site is zero. This is in compliance with the zero-inventory ordering (ZIO) policy adopted by the system.

2.2 Notations

Based on the supply chain system described above, the following are the notations used to model the problem.

Parameters and Notations

| ASi  | setup cost for supplier i ($/setup) |
| AMS  | setup cost for the manufacturer ($/setup) |
| AMO  | ordering cost for the manufacturer ($/order) |
| AR   | ordering cost for the retailer ($/order) |
HSi  annual inventory cost for part i at supplier i ($/unit/year)
HMi  annual inventory cost for part i at the manufacturer’s site ($/unit/year)
HM  annual inventory cost for a finished product at the manufacturer’s site ($/unit/year)
HR  annual inventory cost for a finished product at the retailer’s site ($/unit/year)
PSi  production rate of supplier i (units/year)
PM  production rate of the manufacturer (units/year)
D  demand rate for the finished product (units/year)
Di  demand rate for parts from supplier i (units/year)
QSi  production lot size for supplier i in the original schedule (units)
QMP  production lot size for the manufacturer in the original schedule (units)
QMO  ordering lot size for the manufacturer in the original schedule (units)
QR  ordering lot size for the retailer in the original schedule (units)
BqM  backorder quantity for the manufacturer
BqR  backorder quantity for the retailer
LqM  lost sales quantity for the manufacturer
LqR  lost sales quantity for the retailer
TdM  disruption period for the manufacturer
TdS  disruption period for disrupted supplier
ρ  production uptime for a normal cycle (Q/P)
u  production downtime for a normal cycle
te  start of recovery time window
tf  end of recovery time window
TSi  production cycle time of a normal cycle for supplier i (Q/D)
TM  production cycle time of a normal cycle for the manufacturer (Q/D)
TR  production cycle time of a normal cycle for the retailer (Q/D)
BSD, BM, BR unit backorder cost per unit time for the disrupted supplier, manufacturer and retailer, respectively ($/unit/time)
LSD, LM, LR unit lost sales cost for the disrupted supplier, manufacturer and retailer, respectively ($/unit)
W  warehouse capacity for stage 2 (units)
TSDi  production time for cycle i in the recovery window for the disrupted supplier
TMj  production time for cycle j in the recovery window for the manufacturer
TRk  production time for cycle k in the recovery window for the retailer
Ii  inventory level at the end of cycle i in the recovery window
f1  the penalty function for delay in recovering the original schedule in the first stage
f2  the penalty function for delay in recovering the original schedule in the second stage handled by the first stage
f3  the penalty function for delay in recovering the original schedule in the second stage

Decision Variables
XSDi  production lot size of cycle i in the recovery schedule for the disrupted supplier (units)
XMi  production lot size of cycle i in the recovery schedule for the manufacturer (units)
SMi  the manufacturer’s order lot size of cycle i in the recovery schedule for the disrupted supplier’s parts (units)
SRI  the retailer’s order lot size of cycle i in the recovery schedule for the finished goods (units)
n1  number of cycles in the recovery window for the disrupted supplier and manufacturer
n2  number of cycles in the recovery window for the manufacturer and retailer
z1  number of optimal production lots in the recovery window for the disrupted supplier
z2  number of optimal production lots in the recovery window for the manufacturer
m  number of suppliers in the supply chain system

2.3 Mathematical Representation

In formulating the total costs for recovery, the following cost components are considered: setup/ordering costs, inventory holding costs, penalty costs for late recovery, backorders and lost sales cost. These costs for each entity (supplier, manufacturer and retailer) are totalled in order to find the total cost of the entire
system. The total recovery cost of the system, TC, for the three-stage supply chain system subject to transportation disruption between the suppliers and the manufacturer (disruption types 1, 2, and 3) can be written as follows:

\[
TC(X_{s0}, S_{s1}, n_i, z, X_{s2}, S_{r1}, n_2, z_2) = \left[ \sum_i A_{s0} \cdot z_i + H_{s0} \left( \frac{Q_{s0}^2}{P_{s0}} + \sum_i \left( I_{s1} + \frac{1}{2} X_{s2} \right) \frac{X_{s0}}{P_{s0}} \right) \right] + f_1(n_2^2) + f_2(n_2^2) + \frac{B_{s0} \cdot T_{s0} \sum_i \left( A_{s0} \cdot z_i + \frac{H_{s0}}{2} (T_{s0} Q_{s0} z_i) \right)}{n_i T_{s0}} + \frac{B_{s0} \cdot T_{s0} \sum_i \left( T_{s0} Q_{s0} z_i \right)}{n_i T_{s0}} + \frac{B_{s0} \cdot T_{s0} \sum_i \left( T_{s0} Q_{s0} z_i \right)}{n_i T_{s0}}
\]

On the other hand, for the transportation disruption between the manufacturer and the retailer (type 4), TC is formulated as

\[
TC(X_{s0}, S_{s1}, n_i, z, X_{s2}, S_{r1}, n_2, z_2) = \left[ \sum_i A_{s0} \cdot z_i + H_{s0} \left( \frac{Q_{s0}^2}{P_{s0}} + \sum_i \left( I_{s1} + \frac{1}{2} X_{s2} \right) \frac{X_{s0}}{P_{s0}} \right) \right] + f_1(n_2^2) + f_2(n_2^2) + \frac{B_{s0} \cdot T_{s0} \sum_i \left( A_{s0} \cdot z_i + \frac{H_{s0}}{2} (T_{s0} Q_{s0} z_i) \right)}{n_i T_{s0}} + \frac{B_{s0} \cdot T_{s0} \sum_i \left( T_{s0} Q_{s0} z_i \right)}{n_i T_{s0}} + \frac{B_{s0} \cdot T_{s0} \sum_i \left( T_{s0} Q_{s0} z_i \right)}{n_i T_{s0}}
\]
The above models can be categorized as constrained integer nonlinear programming models. By solving models (1) and (2) above for the above model subject to the system constraints, the optimal recovery plan can be obtained for the three-stage supply chain system under transportation disruption types 1, 2 and 3, as well as 4, respectively. Nevertheless, as transportation disruption results in supply disruption at the later stages of the SC, the cost functions for the supply disruption at the sites of the manufacturer and retailer, respectively, are provided as follows:

\[ TC_{M1}^2(X_{Mi}, n_2, z_2) = \left( A_{Mi} \cdot (z_1) + \left( H_M \left( \frac{1}{2P_M} q^2 + qT_d + \sum_{i=1}^{n_2} \left( I_{M1} + \frac{1}{2} X_{Mi} \right) \frac{X_{Mi} P_M}{P_M} \right) \right) + f_1(n_2^2) + f_2(n_2^2) \right) + \left( B_{Mi} \left( u + \frac{q}{P_M} + T_d + \frac{Q_{Mi}}{P_M} \right) \frac{Q_{M1}}{2} \right) + (L_M(Lq_R)) \] (3)

\[ TC_{R1}^3(S_{R1}, n_2, z_2) = \left( A_{R1} \cdot (z_1 - 1) + \left( H_R \left( \frac{Q_{R1}}{2} + (S_{R1} - Bq_R)^2 + \sum_{i=1}^{n_2} S_{R1}^2 \right) \right) \right) + f_3(n_2^2) \] (4)

3 Solution Procedure

An algorithm is presented in this section to solve the problem of transportation disruptions. The method is capable of determining the decision variables, which consist of the disrupted supplier’s and manufacturer’s production quantities, as well as the ordering quantities of the manufacturer and retailer, for the optimal recovery schedule in real-time.

Due to the complexity of the problem, it is difficult to obtain an optimal solution for equations (1) and (2) by solving the three-stage SC problem in one optimization model. Therefore, the proposed solution was partitioned into two phases. The first phase will solve the following problem:

\[ TC_1 = \text{Min} \left( TC_{M0} + TC_{M1} \right) \] (5)

The second phase of the solution procedure will solve the following:

\[ TC_2 = \text{Min} \left( TC_{M2} + TC_{R1} \right) \] (6)

The model was divided into two sub-models, which utilized the heuristics T1 and P4 developed in the works by Hishamuddin et al. [15],[16] as the efficient solution technique. The steps in the algorithm for solving the model for transportation disruptions are described here:

Phase I: Production-procurement policy between disrupted supplier and manufacturer
Step 0: Initialize the system parameters
Step 1: If the supplier experiences disruption types 1-3, implement heuristic T1 to solve equation (5) for the decision variables.
Otherwise for disruption type 4, perform Phase II.
Step 2: Calculate TCSI, TCMOi, TCIv.
Step 3: Record the solutions from Phase I.

Phase II: Production-procurement policy between manufacturer and retailer
Step 0: Initialize the system parameters
Step 1: For disruption types 1-3, implement heuristic P4 to solve equation (6) for the decision variables
For disruption type 4, implement heuristic T1 to solve equation (2) for the decision variables
Step 2: Record the solutions from Phase II.
Step 3: Compute the performance measure, TC, namely equation (1) for disruption types 1, 2 and 3, and equation (2) for disruption type 4.

3.1 Sensitivity Analysis

A sensitivity study was carried out to explore the effects of transportation disruptions on the solution and total system cost functions. The impact of the supplier’s system parameters, setup and inventory holding costs, particularly \((A_{Si}, H_{Si})\), were examined by doubling these values. The experiment was designed for a number of scenarios for the system. These designs were created by setting different combinations of supplier parameters. Different values of the setup cost and inventory holding cost for the suppliers were combined to obtain varying supplier lot sizes, \(Q_{Si}\), consisting of high, medium and low lot sizes. Table 3 depicts the suppliers’ input parameters for the experiment. The other parameters, if not varied, are fixed and equal to that of Table 2.

**Table 2: Input parameters for numerical example**

| Cost Unit | Suppliers | | Manufacturers | Retailer |
|-----------|-----------|----------------|------------|
| \(A_{Si}\) | 200 | 400 | 200 | \(A_{MOi}\) | 20 | 50 | 20 | \(A_{MS}\) | 200 | \(A_{R}\) | 20 |
| \(H_{Si}\) | 1.2 | 1.2 | 4 | \(H_{M}\) | 1.8 | \(H_{R}\) | 2 |
| \(H_{MOi}\) | 1.8 | Ta | 5 | \(P_{S}\) | 5000000 | 5000000 | 5000000 | \(P_{M}\) | 500000 |
| \(D_{Si}\) | 4000000 | 4000000 | 4000000 | \(D_{MOi}\) | 4000000 | \(D_{R}\) | 4000000 |
| \(Q_{Si}\) | 25252.3 | 36115.8 | 14650.4 | \(Q_{MOi}\) | 22619.19 | \(Q_{R}\) | 22619.19 |
| \(T_{d}^{S}\) | 0.003 | 0.003 | 0.003 | \(T_{d}^{M}\) | 0.003 | \(T_{d}^{R}\) | 0.003 |
| \(B_{SD}\) | 1 | 1 | 1 | \(B_{M}\) | 1 | \(B_{R}\) | 1 |
| \(L_{SD}\) | 15 | 15 | 15 | \(L_{M}\) | 15 | \(L_{R}\) | 15 |

**Table 3: Input parameters for suppliers**

<table>
<thead>
<tr>
<th>Q Size</th>
<th>(A_{Si})</th>
<th>(A_{MOi})</th>
<th>(H_{Si})</th>
<th>(H_{MOi})</th>
<th>(P_{S})</th>
<th>(D_{Si})</th>
<th>(Q_{Si})</th>
</tr>
</thead>
<tbody>
<tr>
<td>High Q</td>
<td>400</td>
<td>50</td>
<td>1.2</td>
<td>1.8</td>
<td>5000000</td>
<td>4000000</td>
<td>36115.8</td>
</tr>
<tr>
<td>Medium Q</td>
<td>200</td>
<td>20</td>
<td>1.2</td>
<td>1.8</td>
<td>5000000</td>
<td>4000000</td>
<td>25252.3</td>
</tr>
<tr>
<td>Low Q</td>
<td>200</td>
<td>20</td>
<td>4</td>
<td>5</td>
<td>5000000</td>
<td>4000000</td>
<td>14650.5</td>
</tr>
</tbody>
</table>

**Table 4: Design of the experiment with a total of 14 scenarios**

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Case 1 ((LSD, M, R=15, BSD, M, R=1))</th>
<th>Scenario</th>
<th>Case 2 ((LSD, M, R=2, BSD, M, R=1))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Supplier 1</td>
<td>Supplier 2</td>
<td>Supplier 3</td>
<td>Supplier 1</td>
</tr>
<tr>
<td>Scenario 1</td>
<td>High</td>
<td>High</td>
<td>High</td>
</tr>
<tr>
<td>Scenario 2</td>
<td>Low</td>
<td>Low</td>
<td>Low</td>
</tr>
<tr>
<td>Scenario 3</td>
<td>Medi</td>
<td>Medi</td>
<td>Medi</td>
</tr>
<tr>
<td>Scenario 4</td>
<td>um</td>
<td>um</td>
<td>um</td>
</tr>
<tr>
<td>Scenario 5</td>
<td>Medi</td>
<td>um</td>
<td>High</td>
</tr>
<tr>
<td>Scenario 6</td>
<td>um</td>
<td>Medi</td>
<td>um</td>
</tr>
<tr>
<td>Scenario 7</td>
<td>Medi</td>
<td>um</td>
<td>Low</td>
</tr>
</tbody>
</table>
Since the system under consideration had three suppliers, different values of QSi for the suppliers were combined and 7 scenarios were generated. The design of the experiment is shown in Table 4. Each scenario was tested for two cases of lost sales cost, namely low (LSD,MR = 2) and high (LSD,MR = 15), which added up to a total of 14 scenarios. For each scenario, the disruption occurrence was tested at suppliers 1, 2, 3 and at the manufacturer’s site, where the corresponding minimum total cost, which is the performance measure of the system, was obtained. The results of the minimum total cost from these experiments were then examined to discover the influence of various factors on the total inventory system. Furthermore, in the possible event of a disruption, the more critical supplier who deserved the implementation of safety measures was identified, and several managerial insights were proposed.

3.2 Results and Analysis

The results of the sensitivity analysis for the transportation disruption case were slightly different from the supply disruption case in [14]. It can be observed from Figure 1, that the solution was more sensitive to lot size as compared to the cost parameters, setup cost (A) and inventory holding cost (H), for those cases where lost sales cost was high. With the increase in lot size, the TC value increased. This can be explained by the fact that when the goods are damaged in the event of a transportation disruption, the high cost of lost sales outweighs all other costs.

![Figure 1](image1.png)  
*Figure 1: Histogram for total cost of different scenarios of transportation disruptions in cases of high L.*

![Figure 2](image2.png)  
*Figure 2: Histogram for total cost of different scenarios of transportation disruptions in cases of low L.*
However, when lost sales cost was low, the solution was seen to be more dependent on \( A \) and \( H \), as seen in Figure 2. As the lost sales quantity was large for the overall case of transportation disruption with a damaged lot, the solution was found to be more sensitive to the lost sales cost parameter, \( L \). Additionally, an overall conclusion could be made from the analysis in which lost sales cost was found to be more sensitive to the total costs for transportation disruptions as opposed to supply disruptions.

4 Conclusion

This paper presented an optimization approach for the problem of rescheduling a three-stage supply chain system with multiple suppliers. Based on the optimization technique, the effects of disruptions over time in the multi-echelon supply chain and the optimal recovery properties of the supply chain network were analysed. In addition, the effects of disruptions at different locations of the supply chain were examined, particularly with regard to system costs and stockout levels. Furthermore, suggestions on the prioritization of disruption mitigation strategies for the suppliers were proposed.

An analysis of the experimental results can provide useful managerial insights for a manufacturing firm. Firstly, the lost sales cost parameter has a major influence on total recovery costs in the event of a transportation disruption where goods are damaged. Therefore, it is critical to have a good and reliable transportation backup system for suppliers with high lost sales cost. It is also crucial to have a backup inventory for the parts from suppliers that produce large lot sizes. This is due to the higher recovery costs incurred by the system when the lot from such suppliers gets damaged in a transportation disruption. There are several directions in which this research may continue. Extending the model to a multi-echelon supply chain with multiple retailers is a worthwhile extension that is currently being undertaken.

References