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A two-stage stochastic model for the winner determination problem in transportation procurement auctions

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Abstract. We propose a service-oriented combinatorial auction mechanism in an uncertain context where a shipper needs to outsource its transport operations to external carriers. Uncertainty is on shipper demand, carrier capacity and carrier lead time. A two-stage stochastic formulation is proposed to model the winner determination problem. A Monte Carlo approach combined with the sample average approximation method is proposed to solve the problem. Our preliminary results prove the efficiency and the relevance of the proposed approach.

Key words: 2-stage stochastic programming, transportation procurement, winner determination problem

1 Introduction

We consider a combinatorial auction for transportation services procurement where a shipper needs to outsource some or a part of its transport operations to external carriers. Carriers are selected at the strategic/tactical level and long-term relationships are build with them for the upcoming one to three years. The shipper, running the auction, presents its transportation needs (shipments) to the set of carriers invited to participate into the auction. Each carrier submits one or multiple bids on the contracts it is interested in. A combinatorial auction enables a carrier to express its interests for a package of contracts in the same bid, known as a combinatorial bid. After receiving all carriers' bids, the shipper has to determine the auction winners by solving the so-called winner determination problem (WDP). Our paper addresses the WDP under uncertainty.

The selection decisions being made at the strategic/tactical level, a number of problem parameters are therefore not known with certainty. In this paper, we propose a service-oriented combinatorial auction mechanism under shipper demand, carrier capacity and carrier lead time uncertainties. To the best of our knowledge, all previous research on stochastic WDP for TL transportation services procurement address uncertainty on shipment volumes only and minimize the shipper transportation costs [1] [2] [3] [4].

The remainder of the paper is as follows. Section 2 describes the auction environment addressed. Section 3 presents the deterministic and the two-stage stochastic model proposed for the WDP under study. Section 4 presents the proposed solution approach. Preliminary results are discussed in Section 5.

2 Auction environment

We consider a one-sided reverse auction with a single shipper acting as the auctioneer and a set of competing carriers acting as bidders. A shipper request is defined by an origin-destination pair (a lane), and minimum and maximum volumes to be transported on each lane. A "revenue" parameter is associated to each lane to represent the unit net revenue, without considering transport operations, resulting from shipping products on this lane. Each carrier's bid gathers the set of lanes it offers to serve, the price asked for shipping one volume unit on each lane, the lead time offered for each lane, and some bounds on the minimum and maximum volumes to transport.

Combinatorial bidding assumes all-or-nothing bids. In other words, if a carrier wins a bid, it must ensure the service for all the lanes covered by this bid. Moreover, we consider a XOR bidding language [5]. That is, a carrier can submit any number of bids it wants but in the final allocation it can be awarded at most one bid. It is also assumed that each lane can be served by at most one participating carrier. Other additional constraints are considered from the shipper perspective to better manage its relationship with the participating carriers. Hence, the shipper imposes minimum and maximum volumes to be awarded to each carrier and limits the total number of winning carriers.

In the problem considered, bids are given data and their construction are not addressed. The paper rather focuses on the winner determination problem. The objective is to choose bids and associated volumes that maximize the shipper profit and satisfy the shipping minimum and maximum demands on shipper's lanes. The shipper profit concerns the transport operations only, assuming thus that all other operations costs are known and already deduced from sell prices. Moreover, a carrier is assumed to submit in each bid covering a lane a promised shipping time. This time is then compared to the observed time during operations. The difference between these two times is penalized in the objective function in order to shape the importance of the service level in this problem. In case carriers' bids are not able to satisfy all lanes' demands, the shipper has the possibility to call a carrier from the spot market to ensure the shipment of the remaining unsatisfied demands. We assume that it is always more expensive for the shipper to satisfy the demand by the spot carrier than by negotiating contracts with the carriers participating in the auction.Hereafter, we recall the notation and terminology used and that we will adopted throughout the paper.

R	set of carriers (bidders)
L	set of lanes
d_l^{min}	minimum demand on lane $l \in L$
d_l^{max}	maximum demand on lane $l \in L$
p_l	sell price of one unit of volume on lane l minus unit costs except transport
B_r	set of bids of carrier $r \in R$
$\mathcal{L}_{rb}$	set of lanes that carrier r offers to serve in bid b
a_{rb}^l	a constant parameter: $a_{rb}^l = 1$ if $l \in \mathcal{L}_{rb}$; $a_{rb}^l = 0$, otherwise
LV_{rb}	minimum volume guaranteed to the carrier if bid b wins
UV_{rb}	maximum volume that the carrier can ship if bid b wins
c_{rb}	price asked by carrier r in bid b for transporting one unit volume on each lane $l \in \mathcal{L}_{rb}$
RT_{rb}^{l*}	lead time promised by carrier r in its bid b for lane $l \in \mathcal{L}_{rb}$
RT_{rb}^{*}	set of lead times associated with the bid b of carrier r:$RT_{rb}^* = \{RT_{rb}^{l*}; l \in L_{rb}\}$
RT_{rb}^{l}	lead time observed for lane $l \in \mathcal{L}_{rb}$ by carrier r winning bid b
u_l	unit delay/advance cost associated with one unit shipped on lane l
u_{rb}^l	unit penalty cost associated with lane l with regard to bid b submitted by carrier r
ce_l	cost of shipping one unit volume on lane l by a spot carrier
q_r	minimum volume to allocate to carrier r if it wins
Q_r	maximum volume to allocate to carrier r if it wins
N_{min}	minimum number of winning carriers
N_{max}	maximum number of winning carriers

3 Modelling Approach

3.1 Deterministic Model

We consider the following decision variables:

- $x_{rb} = 1$ if bid b wins, $x_{rb} = 0$, otherwise; $\forall r \in R, b \in B^r$.
- y_{rbl} = the volume allocated to bid b submitted by carrier r on lane l.
- e_l= the volume on lane l assigned to the spot market.

The deterministic WDP is formulated as follows:

$$\max_{x,y,e} \sum_{l\in L}\sum_{r\in R}\sum_{b\in B_r}(p_l - u_{rb}^l)a_{rb}^l y_{rbl} - \sum_{l\in L}\sum_{r\in R}\sum_{b\in B_r} c_{rb}y_{rbl} + \sum_{l\in L}(p_l - ce_l)e_l \quad (1)$$

$$\sum_{r\in R}\sum_{b\in B_r} a_{rb}^l y_{rbl} + e_l \geq d_l^{min} \qquad l \in L \quad (2)$$

$$\sum_{r\in R}\sum_{b\in B_r} a_{rb}^l y_{rbl} + e_l \leq d_l^{max} \qquad l \in L \quad (3)$$

$$LV_{rb}x_{rb} \leq y_{rbl} \leq UV_{rb}x_{rb} \qquad r \in R, b \in B_r, l \in L_{B_r} \quad (4)$$

$$\sum_{b\in B_r} x_{rb} \leq 1 \qquad r \in R \quad (5)$$

$$\sum_{r\in R}\sum_{b\in B_r} a^l_{rb}x_{rb} \leq 1 \qquad l \in L \tag{6}$$

$$N_{min} \leq \sum_{r\in R}\sum_{b\in B_r} x_{rb} \leq N_{max} \tag{7}$$

$$q_r \sum_{b\in B_r} x_{rb} \leq \sum_{b\in B_r}\sum_{l\in L_{B_r}} y_{rbl} \leq Q_r \sum_{b\in B_r} x_{rb} \qquad r \in R \tag{8}$$

$$x_{rb} \in \{0,1\}, y_{rbl} \geq 0, e_l \geq 0 \qquad r \in R, b \in B_r, l \in L \tag{9}$$

Objective function (1) maximizes the shipper profit associated with transport operations taking into account the carrier performance in terms of lead times along the horizon plan. Constraints (2) and (3) ensure that the minimum and maximum demands on each lane are satisfied either by winning bids or the spot market. Constraints (4) restrict the volume allocated to each lane of a winning bid to the minimum and maximum volumes imposed by the carrier in its bid. Constraints (5) model the XOR bidding language. Constraints (6) ensure that each lane is served by at most one winning bid. Restrictions on the minimum and maximum number of winning carriers are ensured by constraint (7). Constraints (8) limit the total volume allocated to a winning carrier to the minimum and maximum volumes imposed by the shipper. Constraints (9) specify the nature of the decision variables used.

3.2 Scenario-based Stochastic Model

In this paper, three parameters are assumed uncertain: the shipper demand levels (d^{min}_l and d^{max}_l), the carriers capacity (UV_{rb}) and the carriers service time (RT^{l*}_{rb}). We propose a two-stage stochastic model where: the first-stage variables are the bid-allocation decisions (x_{rb}) and the recourse variables are the volume allocation decisions (y_{rbl} and e_l). It is assumed that the probability distributions of all the uncertain parameters are available, based on historical data, and can thus be used to generate plausible future scenarios. Scenarios of demand and lead times are generated over a planning horizon T discretized into equal periods lengths t (on a daily basis, for example). In the following, we denote Ω the set of all plausible future scenarios. For a scenario $\omega \in \Omega$, the following instance of the demands, capacities and service times are obtained: $d^{min}_{lt}(w)$, $d^{max}_{lt}(w)$, $RT^l_{rbt}(w)$ and $UV_{rb}(w)$.

Based on this, the two-stage stochastic WDP can be formulated as follows:

$$\max_{x,y,e}[\ \mathbb{E}_{\omega\in\Omega}\ \{\ \sum_{t\in T}\sum_{l\in L}\sum_{r\in R}\sum_{b\in B_r}(p_l - u^l_{rbt}(w))a^l_{rb}y_{rblt}(w) - \\ \sum_{t\in T}\sum_{l\in L}\sum_{r\in R}\sum_{b\in B_r} c_{rb}y_{rblt}(w) + \sum_{t\in T}\sum_{l\in L}(p_l - ce_l)e_{lt}(w)\ \}\ ] \tag{10}$$

$$\sum_{r\in R}\sum_{b\in B_r} a^l_{rb}y_{rblt}(w) + e_{lt}(w) \geq d^{min}_{lt}(w) \qquad l \in L, t \in T, w \in \Omega \tag{11}$$

$$\sum_{r\in R}\sum_{b\in B_r} a^l_{rb} y_{rblt}(w) + e_{lt}(w) \leq d^{max}_{lt}(w) \qquad l\in L, t\in T, w\in \Omega \tag{12}$$

$$LV_{rb}x_{rb} \leq \sum_{t\in T} y_{rblt}(w) \leq UV_{rb}(w)x_{rb} \qquad r\in R, b\in B_r, w\in\Omega \tag{13}$$

$$\sum_{b\in B_r} x_{rb} \leq 1 \qquad r\in R \tag{14}$$

$$\sum_{r\in R}\sum_{b\in B_r} a^l_{rb}x_{rb} \leq 1 \qquad l\in L \tag{15}$$

$$N_{min} \leq \sum_{r\in R}\sum_{b\in B_r} x_{rb} \leq N_{max} \tag{16}$$

$$q_r\sum_{b\in B_r} x_{rb} \leq \sum_{t\in T}\sum_{b\in B_r} y_{rblt}(w) \leq Q_r\sum_{b\in B_r} x_{rb} \qquad r\in R, w\in\Omega \tag{17}$$

$$x_{rb}\in\{0,1\}, y_{rbt}(w)\geq 0, e_{lt}(w)\geq 0 \quad r\in R, b\in B_r, l\in L, t\in T, w\in\Omega \tag{18}$$

4 Solution Approach

The previous section assumed that the uncertainty is captured through a set of scenarios, whereas deterministic modelling relies on point estimates of parameters. Based on the uncertainty characterization, a scenario is defined as a compound event which is the result of the juxtaposition of random processes related to the demand, the capacity and the shipments lead-time. Let $\Omega^N \subset \Omega$ denotes a sample of N scenarios. First, we assume that the minimum demand on lane l during a given period t is a random variable d^{min}_{lt} with a log-normal distribution function $F^{d^{min}}(d^{min}_{lt})$, a mean value $\mu^{d^{min}}_l$, and standard deviation $\sigma^{d^{min}}_l$. Similarly, the maximum demand on lane l during period t is a random variable d^{max}_{lt} with a log-normal distribution function $F^{d^{max}}(d^{max}_{lt})$, a mean value $\mu^{d^{max}}_l$, and standard deviation $\sigma^{d^{max}}_l$. Second, we assume that the maximum capacity of the carrier available over the whole planning horizon T on any lane (UV_{rb}) follows a stochastic process depending on the number of effective available trucks. We consider that some trucks may become unavailable during the planning horizon due to some endogenous or exogenous events such as failures, accidents, maintenance operations, etc. Accordingly, the capacity of the carrier is computed as:

$$UV_{rb} = \overline{UV}_{rb} - \hat{\rho}\lambda_r,$$

where λ_r is a random variable that follows a Poisson distribution function $F^{\lambda}(\lambda_r)$ with a mean number $\overline{\lambda}_r$ representing the number of unavailable trucks over the planning horizon. $\hat{\rho}$ estimates the loss in capacity due to a truck unavailability. It is computed as the product of the trucks loading capacity and the average number of days of trucks unavailability. Finally, on a day to day basis, on-time delivery could be altered due to exogenous factors such as congestions, accidents, customs delays. Thus, the carrier r effective lead time for a given lane l (RT^l_{rbt}) is a random variable, and follows a discrete triangular distribution function $F^{RT}(RT^l_{rbt})$ with minimum value γ_{rl}, maximum value $\overline{\gamma_{rl}}$ and modal value $\dot{\gamma_{rl}}$.

Given these stochastic processes and using a Monte Carlo procedure, we generate a business scenario $\omega \in \Omega$ over the planning horizon T. To do this, we generate independent pseudo-random numbers uniformly distributed on the interval $[0, 1]$, and use them to compute the inverse of the distributions of all the random variable considered.

The next step is the optimization of the mathematical models. [6] proposed the sample average approximation (SAA) technique that consists in replacing the set of all plausible scenarios Ω by a sample Ω^N in the stochastic model (10)-(18) and solving the equivalent deterministic MIP. Accordingly, one should consider solving the following SAA program for Ω^N :

$$\max_{x,y,e} \frac{1}{N} \sum_{\omega \in \Omega^N} \sum_{t \in T} \sum_{l \in L} \sum_{r \in R} \sum_{b \in B_r} (p_l - u^l_{rbt}(w)) a^l_{rb} y_{rblt}(w) \qquad (19)$$

$$-\frac{1}{N} \sum_{\omega \in \Omega^N} \sum_{t \in T} \sum_{l \in L} \sum_{r \in R} \sum_{b \in B_r} c_{rb} y_{rblt}(w) + \frac{1}{N} \sum_{\omega \in \Omega^N} \sum_{t \in T} \sum_{l \in L} (p_l - ce_l) e_{lt}(w) \qquad (20)$$

subject to constraints (11)-(18), $\omega \in \Omega^N$.

5 Preliminary Results and Future Work

5.1 Plan of experiments

We generate an illustrative instance where the three main dimensions are set: the number of lanes $|L| = 30$, the number of carriers $|R| = 10$, and the number of bids submitted by each carrier $|B| = 5$. We assume that the carriers submit the same number of bids (i.e., $|B_r| = |B| = 5, \ \forall r \in R$).

The lanes considered are of three types according to their demand level: 15% of the lanes have low demand, 65% moderate demand, and 20% high demand. For each category, the minimum demand and maximum demand are given in Table 1. To cover a more realistic business context, we also partitioned the pool of carriers into three categories according to their capacity as proposed in the bids (i.e. $UV_{rb}, b \in B_r$): large-size carriers (they represent about 20% of the participants), medium-size carriers (about 65%), and small-size carriers (about 15%). The minimum and maximum number of winners are fixed to $N_{min} = 2$ and $N_{max} = 5$ (which represent 50% of the participants). The minimum volume (q_r) and the maximum volume (Q_r) allowed by the shipper for a carrier of each category are given in Table 2. The unitary sales revenue is generated in the interval [100, 170], and the unitary cost of the spot market is uniformly generated in the interval [50, 100].

Demand level	$\mu_l^{d^{min}}$	$\mu_l^{d^{max}}$	$\sigma_l^{d^{min}} = \sigma_l^{d^{max}}$
Low	[4800, 7200]	[5280, 7920]	
Moderate	[7200, 9600]	[8640, 11520]	[10%, 15%] $\mu_l^{d^{min}}$
High	[9600, 12000]	[12000, 15000]	

Table 1: Demand characteristics for lanes $l \in L$

Carrier size	q_r	Q_r	LB_{rb}	$\overline{B}_{rb}$
Small	$\frac{4800}{N_{max}}$	10 800	[1200,1920]	[10080, 11520]
Medium	$\frac{7200}{N_{max}}$	14 400	[1800,2880]	[13440, 15360]
Large	$\frac{9600}{N_{max}}$	18 000	[2400, 3840]	[16800, 19200]

Table 2: Shipper data on volume bounds per carrier category

5.2 Results discussion

Given the instance defined above, the deterministic model (1)-(9) and the stochastic model (10)-(18) are optimized using the Cplex commercial solver. A sample of 30 scenarios is produced with the MonteCarlo procedure. Recall that since the model is multi-period (12 periods are considered), when 30 scenarios are generated, this gives 30 x 12 periods instantiation of the stochastic process. Table 3 reports some characteristics of the stochastic and the deterministic solutions for a typical run. Observe that a run corresponds to fixed values of the probability distribution parameters of the random variables (demand, capacity and lead times) from which the scenarios are generated.

	Deterministic solution	Stochastic solution
Winning bids (total number)	3-13-21-42 (4)	3-13-21 (3)
Winning carriers (total number)	0-2-4-8 (4)	0-2-4 (3)
Number of contracts spot market	11	16

Table 3: Solutions characteristics

The results of Table 3 show that the deterministic model tends to produce solutions that involve more winning bids (and consequently more carriers). This is probably due to the spot market recourse that is conversely higher for the stochastic solution (about 30% more). These results reflect the flexibility provided by the stochastic modelling approach where the spot market prevails upon inflexible long-term contracts with possibly unreliable carriers.

Table 4 compares the quality of the stochastic and the deterministic solutions for 5 runs of the problem instance based on a posteriori evaluation on a large sample of 500 scenarios.

Gap (%)	Run 1	Run 2	Run 3	Run 4	Run 5
Expected total profit	0.41	0.00	1.26	1.57	0.36
Expected total cost	-3.96	0.00	-8.93	-14.73	-11.80
Delay penalties	-9.03	0.00	-9.43	-20.54	-8.92
Auction transportation costs	-12.22	0.00	-11.80	-28.24	-17.36
Spot market transportation costs	17.29	0.00	12.30	34.05	14.48

Table 4: Comparative results of the stochastic and the deterministic solutions

The results of Table 4 show that the stochastic solution results in a total profit that is either equal or is larger than the deterministic solutions. The savings in total costs is significant for 4 runs out of 5 and reaches 14.73% (Run 4). This is essentially due to the savings obtained for delay penalties with the stochastic solution (as depicted in the row Delay Penalties). One could observe that the transportation cost paid to the spot market is much larger for the stochastic solution where as the transportation costs paid to the auction winners is considerably smaller. This confirms our observation that the stochastic solution is more flexible and calls for the spot market more frequently to overcome the uncertainty in carriers performance with regard to lead times.

These results are encouraging towards the inclusion of a more comprehensive uncertainty set. Obviously, more experiments are required to confirm our preliminary results. Moreover, even with the power of current solvers, the scenario samples, which can be used for large size problems, are still relatively moderate and thus more sophisticated optimization approaches must be developed.

References

1. Ma, Z., Kwon, R.H., Lee, C.G.: A stochastic programming winner determination model for truckload procurement under shipment uncertainty. Transportation Research Part E: Logistics and Transportation Review **46**(1) (2010) 49 – 60
2. Remli, N., Rekik, M.: A robust winner determination problem for combinatorial transportation auctions under uncertain shipment volumes. Transportation Research Part C: Emerging Technologies **35** (2013) 204 – 217
3. Zhang, B., Ding, H., Li, H., Wang, W., Yao, T.: A sampling-based stochastic winner determination model for truckload service procurement. Networks and Spatial Economics **14**(2) (2014) 159–181
4. Zhang, B., Yao, T., Friesz, T., Sun, Y.: A tractable two-stage robust winner determination model for truckload service procurement via combinatorial auctions. Transportation Resaerch Part B **78** (2015) 16–31
5. Nisan, N.: Bidding languages for combinatorial auctions. In P. Cramton, Y. Shoham, R.S., ed.: Combinatorial Auctions. MIT Press (2006) 215–232
6. Shapiro, A.: Stochastic programming approach to optimization under uncertainty. Math Program Ser A **112**(1) (2008) 183–220