A New Multi-Criteria ABC Inventory Classification Model Based on a Simplified Electre III Method and the Continuous Variable Neighborhood Search

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Abstract. . ABC analysis is one of the most widely used techniques in inventory management to classify items into three ordered categories called A, B and C, where category A contains the most important items and category C includes the least important ones. The ABC classification of inventory items is based on their performance expressed by weighted scores. In this paper, a new ABC classification model - based on a simplified ELECTRE III method - will be used as an aggregation procedure to compute the items scores. Since the application of the simplified ELECTRE III requires the knowledge of some parameter values (e.g. indifference, preference and veto thresholds), the Continuous Variable Neighborhood Search (CVNS) metaheuristic is used for their estimation. To analyze the performance of the proposed classification model with respect to some existing models, a benchmark dataset from an Hospital Respiratory Therapy Unit (HRTU) is used

Keywords: ELECTRE III, Variable Neighbourhood Search, MCDM, ABC classification, Inventory Management, Outranking method

1 Introduction

ABC analysis is a well-known technique used in inventory management to categorize inventory items into three ordered and predefined categories: category A contains the most valuable items, category B includes the moderately valuable items and category C contains the least valuable ones. The main aim the ABC classification is to manage in an effective way a large number of items by determining what inventory control policy should be adopted for each category of items. Traditional ABC analysis uses only the annual dollar usage (ADU) criterion to classify inventory items into one of the three categories. Nevertheless, recent literature has shown that many other relevant criteria should be considered in the item classification process (e.g. ordering cost, criticality, lead time, obsolescence, substitutability, order size requirement, etc.). Hence, in a multicriteria framework, the performance of each inventory item - on which the ABC classification is based - is measured by an overall weighted score that combines the item evaluation on the different criteria and the criteria weights. Once these scores are computed, the items are then ranked in a descending order according to their performance. Finally, the different categories are built as follows: the first ranked items (about 5%-10% of the item number) are classified in category A, the last ranked items (about 50%-70% of the item number) are classified in category C and category B will include the remaining items (about 20%-30% of the item number).
In any ABC classification model, three main families of techniques are often used: Mathematical Programming (MP) techniques, Artificial Intelligence (AI) techniques and Multi-Criteria Decision Making (MCDM) techniques. In addition, two main topics are usually tackled by these models: (i) The development of an aggregation procedure to compute the weighted item score, and (ii) The elicitation of the parameters of the aggregation procedure.
Classification models based on MP techniques propose weighted linear or nonlinear optimization models in order to compute the global score of each item. The main aim of these optimization models is to generate a vector or a matrix of weights that maximizes the performance of each item expressed by a weighted global score. Ramanathan [1] was the first to develop a linear optimization model for ABC inventory classification

with multiple criteria. This model was then extended by different authors: Zhou and Fan [2], Ng[3], Hadi [4] and Chen [5]. The main weakness of classification models based on MP techniques is the high number of optimization models to be solved when the number of inventory items is large. Classification models based on AI techniques propose meta-heuristics such as Genetic Algorithm [6], Particle Swarm Optimization [7] and Simulated Annealing [8] to estimate the criteria weights. Then, these weights are combined with the item evaluation on the different criteria - by using some aggregation rules (e.g. Weighted Sum) - to compute the weighted score for each item. Other AI-based techniques including Artificial Neural Network (ANN) [9] cluster analysis, support vector machines (SVMs), Back propagation networks (BPN), and the k-nearest neighbor (KNN) algorithm were also proposed [10] . The use of this type of classification models assumes the availability of a training dataset of pre-assigned items to perform the learning process.

Classification models based on MCDM techniques propose a two-steps methodology to classify inventory items into ABC categories. In the first step, an MCDM method - essentially the Analytic Hierarchy Process (AHP)- is applied once to compute the criteria weights. In the second step, an aggregation rule is used to compute the global score of each inventory item. The models proposed by Flores et al[11], Partovi and Burton [12] and Battacharya et al. [13] are some typical MCDM based classification models. The MCDM based classification models have the advantage of incorporating explicitly human judgments, considering conflicting criteria and dealing with data obtained on heterogeneous measurement scales (i.e. quantitative and qualitative).

In this paper, we capitalize the benefits of both AI and MCDM techniques to build a new ABC classification model. In this model an MCDM method - called ELECTRE III (ELimination Et Choix Traduisant la REalité) - is used as aggregation procedure to compute the weighted score of each item. However, since the application of ELECTRE III method requires the knowledge of some parameter values such as weights, indifference, preference and veto thresholds, the Entropy method and the Continuous Variable Neighborhood Search (CVNS) will be used to estimate these parameters. To analyze the performance of the proposed classification model with respect to some existing models, a benchmark dataset from an Hospital Respiratory Therapy Unit (HRTU) is used.

Compared to the existing classification models, three main advantages of our proposed model should be stated. First, the a priori determination of the criteria weights by using the entropy method is much easier than the use of the Analytic Hierarchy Process (AHP), the most applied criteria weighting technique in the existing classification models. Indeed, two main criticisms may be addressed to the AHP method: (i) the number of pairwise comparisons increases rapidly when the number of criteria and/or items is large and (ii) it is very demanding for the decision maker to specify by using Saaty scale how much a criterion is more important than another criterion. Second, despite the benefits of MCDM methods - such as ELECTRE III - their hybridization with metaheuristics is rarely tackled in the literature. Third, contrary to our proposed model, most of the existing classification models produce the ABC classifications of items without considering any inventory control policy. In fact, the classification model developed in this paper will produce an ABC classification of inventory items that minimizes an inventory cost function called Total Relevant Cost (TRC).

The outline of the paper is as follows. Section 2 introduces the theoretical background of the proposed classification model. The general framework of the proposed model is presented in section 3. In section 4, the computational results of the proposed model and some other existing classification models are discussed and compared. Finally, conclusions and perspectives for further research are reported in section 5.

2 Theoretical background

In this section, the main components of the proposed model will be detailed: the ELECTRE III, the Continuous Variable Neighbourhood Search (CVNS) and the Entropy method.

2.1 ELECTRE III method

ELECTRE III [14] is a multi-criteria decision aiding method which uses pairwise comparisons of items in order to rank them by considering incommensurable and conflicting criteria. ELECTRE III method is a two-step procedure. In the first step a valued outranking relation is constructed for each pair of items a and b by measuring the credibility degree of the following statement : *"item a is at least as good as item b"* or shortly *"a outranks b"* . In the second step, these valued outranking relations are exploited in order to produce a ranking of items.

2.1.1 The construction of valued outranking relation

To measure the credibility degree of the statement "a outranks b" , denoted by $\sigma(a,b)$ four steps should be followed:

1. Compute, for each criterion j and for each pair of items a and b, a partial concordance index, denoted by $C_j(a,b)$, in the following manner:

$$C_j(a,b) = \begin{cases} 0 \quad \text{if} \quad g_j(b) - g_j(a) \geq p_j \\ 1 \quad \text{if} \quad g_j(b) - g_j(a) \leq q_j \\ \frac{p_j + g_j(a) - g_j(b)}{p_j - q_j} \quad \text{otherwise} \end{cases} \tag{1}$$

   where $g_j(*)$ is the evaluation of item $(*)$ according to the criterion j, q_j and p_j are respectively the indifference and the preference thresholds of the criterion j. The indifference threshold represents the highest difference between two evaluations according to the same criterion for which the decision-maker is incapable to make a clear choice, given that everything is the same otherwise. The preference threshold represents the smallest difference between two evaluations according to the same criterion for which the decision-maker is able to make a clear preference for one, given that everything is the same otherwise.

2. Compute for each pair of items a and b the global concordance index, denoted by $C(a,b)$, in the following manner:

$$C(a,b) = \frac{\sum_{j=1}^{n} w_j \times C_j(a,b)}{\sum_{j=1}^{n} w_j} \tag{2}$$

   where w_j is the relative importance of criterion j . The global concordance index $C(a,b)$ is a measure of the strength of arguments which agree with the statement " a outranks b ".

3. Compute, for each criterion j and for each pair of items a and b, a partial discordance index, denoted by $D_j(a,b)$, in the following manner:

$$D_j(a,b) = \begin{cases} 1 \quad \text{if} \quad g_j(b) - g_j(a) \geq v_j \\ 0 \quad \text{if} \quad g_j(b) - g_j(a) \leq p_j \\ \frac{g_j(b) - g_j(a) - p_j}{v_j - p_j} \quad \text{otherwise} \end{cases} \tag{3}$$

   where v_j is the veto threshold which represents the limit of tolerance that decision-makers are willing to accept for any compensation. In other words, they may refuse to choose an item a over an item b if the performance of a is higher than the performance of b by at least V_j. The partial discordance index $D_j(a,b)$ is a measure of the strength of arguments which disagree with the statement a outranks b at the level of criterion j.

4. Compute for each pair of items a and b, the credibility degree of the statement "a outranks b":

$$\sigma(a,b) = \begin{cases} C(a,b) \quad if \quad D_j(a,b) \leq C(a,b) \ \forall j \\ C(a,b) \times \prod_{D_j(a,b) > C(a,b)} \frac{1 - D_j(a,b)}{1 - C(a,b)} \quad \text{otherwise} \end{cases} \tag{4}$$

The credibility index corresponds to the concordance index weakened by possible veto effects.

2.1.2 The exploitation of valued outranking relation

Since the original exploitation procedure of ELECTRE III is not easy to understand by the decision makers due to its complexity, we used the exploitation procedure of PROMETHEE II [15] to generate the global score of each item from the credibility degree $\sigma(a,b)$. Hence, the global score of each item is obtained by proceeding in three steps:

1. Compute for each item a its positive outranking flow as follows (m is the number of items) :

$$\Phi^{+}(a) = \frac{1}{m-1}\sum_{x\neq a}\sigma(a,x) \tag{5}$$

The positive outranking flow expresses how an item outranks all the others. It is its strength. The higher the positive flow, the better the item.

2. Compute for each item a its negative outranking flow as follows (m is the number of items):

$$\Phi^{-}(a) = \frac{1}{m-1}\sum_{x\neq a}\sigma(x,a) \tag{6}$$

The negative outranking flow expresses how an item is outranked by all the others. It is its weakness. The lower the negative flow the better the item.

3. The net outranking flow can then be considered to construct a complete ranking of the items, let:

$$\Phi(a) = \Phi^{+}(a) - \Phi^{-}(a) \tag{7}$$

The net outranking flow is simply the global score of items on which the ABC inventory classification is performed.

Thus, in order to use ELECTRE III and generate such scores, we should specify the following parameters (n is number of criteria):

- The weight vector $w = (w_1; \ldots ;w_n) \geq 0$.
- The vector of indifference thresholds $q = (q_1; \ldots ;q_n) \geq 0$.
- The vector of preference thresholds $p = (p_1; \ldots ;p_n) \geq 0$.
- The vector of veto thresholds $v = (v_1; \ldots ;v_n) \geq 0$.

For this purpose, the Entropy method will be used to determine the criteria weights and the Continuous Variable Neighborhood Search (CVNS) meta-heuristic will be proposed to assess the threshold vectors. These two methods will be detailed in the next subsections.

2.2 The Entropy method to determine the criteria weights

In the literature, many methods have been proposed to determine the criteria weights (e.g. the Analytical Hierarchy Process (AHP) [16], Simos method of card [17], etc.). However, most of these methods are essentially based on the decision-maker subjective judgements. To avoid such subjectivity, we adopt in this paper the entropy method [18] . The entropy is a measure of information uncertainty by using probability theory. This measure shows that the more dispersive the data, the bigger the uncertainty. The Shannon's entropy procedure proceeds in the following four steps to determine the criteria weights:

- *Step 1:* Normalize the evaluations of items for each criterion j, let (n is the number of criteria and m the number of items):

$$P_{ij} = \frac{g_j(a_i)}{\sum_{i=1}^{m} g_j(a_i)}, \quad j = 1,2,...,n \tag{8}$$

The initial data are normalized to eliminate inconsistencies with different measurement units and scales.

- *Step 2:* Compute the entropy measure for each criterion as follows:

$$E_j = -\frac{1}{\ln(m)} \sum_{i=1}^{m} P_{ij} \times \ln(P_{ij}), \quad j = 1,2,...,n \tag{9}$$

- *Step 3:* Define the difference degree for each criterion:

$$G_j = 1 - E_j, \quad j = 1,2,...,n \tag{10}$$

The higher the index G_j is, the more important the criterion j is.

- *Step 4:* Compute the normalized criteria weights:

$$W_j = \frac{G_j}{\sum_{j=1}^{n} G_j}, \quad j = 1,2,...,n \tag{11}$$

The main idea of the Entropy method is that the more the variability of items values according to the criterion j is, the more important the criterion j is.

2.3 Continuous Variable Neighborhood Search to estimate ELECTRE III parameters

Variable Neighbourhood Search (VNS) [19] is a well known meta-heuristics designed for solving optimization problems. The idea of (VNS) is to define a set of neighbourhood structures that can be used in a systematic way to conduct a search through the solution space. The (VNS) meta-heuristic has been applied in many areas, specially for solving combinatorial optimization problems. For an overview of the method and numerous applications, the reader is referred to the reference [19] . The basic steps of (VNS) meta-heuristic are as what follows:

Algorithm 1 Variable Neighborhood Search algorithm

Initialization: Select a set of neighbourhood structures N_k and random distribution for the Shaking step that will be used in the search;
Find an initial solution x; choose a stopping condition;
repeat

1. Set $k \leftarrow 1$;
2. Repeat the following steps until $k > k_{max}$:
   (a) **Shaking:** Generate a point (y) randomly from the k_{th} neighborhood of x, ($y \in N_k(x)$);
   (b) **Local search**:Apply some local search method with (y) as initial solution to obtain a local optimum given by (y');
   (c) **Neighbourhood change**: if this local optimum is better than the incumbent, move there ($x \leftarrow y'$), and continue the search with N1 (k←1); *otherwise,* set k←k+1;

until until stopping condition is met

Since in this paper the parameters to be estimated are continuous (indifference, preference and veto thresholds), we will use the Continuous version of VNS denoted (CVNS). The objective function that will guided the CVNS to find the best values of ELECTRE III parameters is the Total Relevant Cost (TRC) which was suggested in Mohammaditabar et al.[8]. The TRC measures the total cost of the obtained ABC classification by considering the ordering cost for each placed order, the setup cost of each item when it is replenished and the holding cost of carrying items in stock. Hence, the TRC of an ABC classification is computed in the following manner:

$$TRC = \sum_g \left(\frac{\sum_{i \in category(g)} S_i}{T_g} + \frac{1}{2} T_g \sum_{i \in category(g)} D_i h_i \right) \tag{12}$$

where the optimal joint replenishment cycle of category g: $T_g = \sqrt{\frac{2(\sum_{i \in category(g)} S_i)}{\sum_{i \in category(g)} D_i h_i}}$
It is assumed that items classified in the same category g (where g= A, B or C) have the same replenishment cycle. S_i, D_i and h_i are the set up cost, the demand and the holding cost per unit of time of item i respectively.

3 The proposed classification model

The general framework of the proposed model described in *Figure1* is based on three important components: The simplified ELECTRE III method is used to compute the global score of each item, the CVNS is used to estimate the ELECTRE III parameters and the Entropy method is used to compute the criteria weights. The aim of the proposed model is to find a set of values for ELECTRE III parameters that provides the best classification of items, i.e. a classification that minimizes the Total Relevant Cost (TRC). Hence, in each iteration, the CVNS generates a thresholds vector (indifference, preference and veto thresholds for each criterion) in the neighborhood of the current solution. By using the thresholds vector and the criteria weights generated by the Entropy method, ELECTRE III is applied to compute a global score for each item. Based on these scores, an ABC classification of inventory items is generated and evaluated by using the TRC function. If any improvement is reached the algorithm will update his best solution or, in the opposite case, will move to the next neighborhood, i.e. a new thresholds vector. The process is repeated until the stopping condition is met.

4 Computational results

To test the performance of the proposed classification model with respect to some other existing models, a benchmark dataset from a Hospital Respiratory Therapy Unit (HRTU) will be used [2]. This dataset includes 47 inventory items evaluated according to three criteria: (i) the Annual Dollar Usage (ADU), (ii) the Average Unit Cost (AUC) and, (iii) the Lead Time (LT). Once the global weighted scores of items are computed, the three categories of the studied dataset are built in the following manner (categories distribution): the first 10 items are classified in category A, the next 14 items in the ranking are classified in category B and the remaining 23 items are classified in category C. For the implementation of the CVNS algorithm some technical choices have been made:

- *The number of neighbourhood structure* k_{max} was fixed to 6.
- *The neighbourhood structures* are defined by the metric l_2 , i.e. the Euclidian distance $d(x,y) = d(y,x) = \sqrt{\sum_{i=1}^{n} (x_i - y_i)^2}$.
- *Radii* are predefined and should verify the following order $r_1 < \ldots < r_{max}$.
- In *shaking step* we generate a random point from $N_k(x)$ as a starting point for the best improvement *local search method.*
- *The stopping condition* is set to the maximal number of iterations.

By applying the Entropy method we have obtained the following criteria weights vector (w_1 = 0.197, w_2 = 0.712, w_3 = 0.091) which was in concordance with other weight vectors in the literature where the (ADU) is the most important criteria. In order to calculate the total relevant cost we needed to set the values of the setup cost, the demand and the holding cost. For this purpose, we used the same values proposed in [8].
The proposed classification model is compared with five existing models from the literature: R-model [1] ZF-model [2], NG-model [3], H-model [4] and Peer Model [5]. The Total Relevant Cost (TRC) of all tested

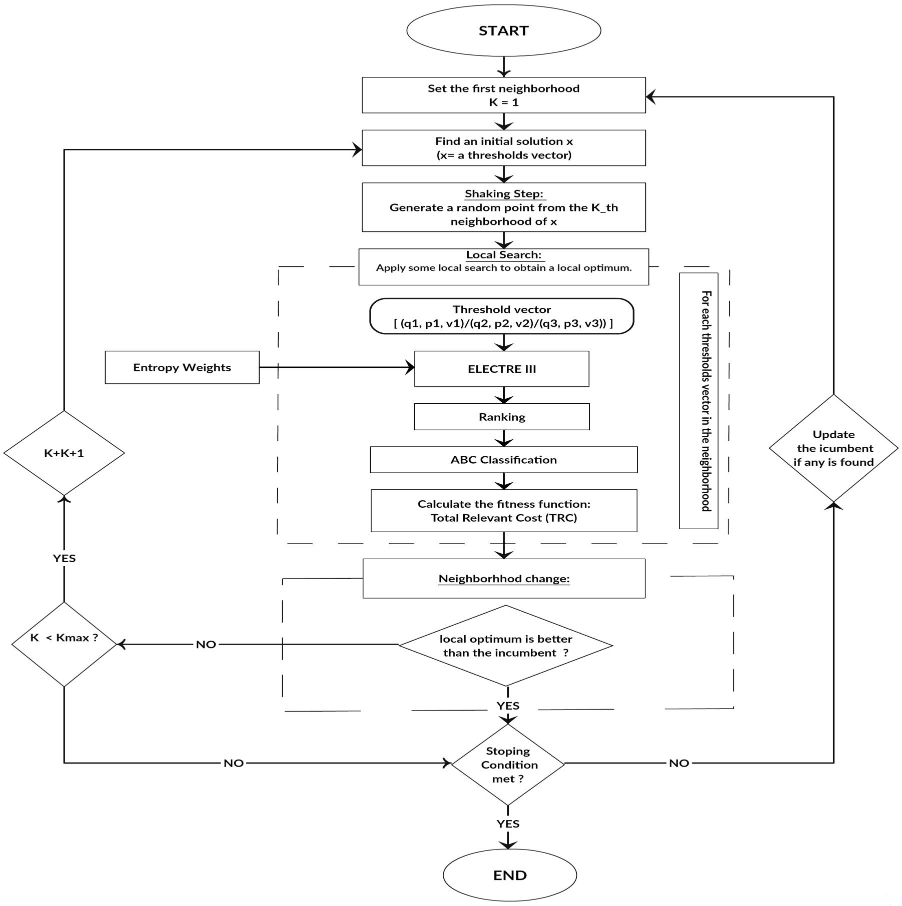

Figure 1: The framework of the proposed model

classification models are presented in Table 1.

After several test runs, the proposed classification model seems to be the most efficient since it obtained the lowest TRC. We have also carried out the pairwise comparison of classification models in order to compute the rate of items that are identically classified by both models. The highest resemblance between the proposed model and all other models is obtained with the NG-model with 60% of items identically classified. This relatively low rate of similarity is quite understandable since our model looks for minimizing an inventory cost function (i.e. TRC) whereas all MP based models seek to optimize, for each item, a weighted score measuring its performance.

Table 1: TRC of the different classification models.

Classification models	R model	ZF model	Peer model	H model	NG model	EIII-VNS worst value	EIII-VNS mean value	EIII-VNS best value
Total Relevant Cost (TRC)	1369,65	1342,16	1334,38	1270,60	1243,55	1127,13	1122,32	**1119,78**

5 Conclusion

In this paper, we have combined the benefit of Multiple Criteria Decision Making (MCDM) and Artificial Intelligence (AI) techniques to propose an hybrid classification model. A simplified version of ELECTRE III (MCDM) is used to compute the weighted global score of each item and the CVNS (AI) is applied to estimate the ELECTRE III parameters namely the indifference, the preference and the veto thresholds. On the other hand, the Entropy method was used to determine the criteria weights. To test the performance of the proposed model with respect some other existing classification model a benchmark dataset from a Hospital Respiratory Therapy Unit (HRTU) is used. We believe that the obtained results are very promising since the proposed model has provided a classification of inventory items that minimizes the total relevant cost (TRC). In further research, we will: (i) test the performance of our model on real world dataset, (ii) consider the criteria weights in the optimization process of the CVNS, (iii) consider quantitative and qualitative criteria in order to exploit the strengths of MCDM methods and, (iv) consider other cost functions such as that suggested by Babai et al.[20].

References

[1] Ramanathan, R.: ABC inventory classification with multiple-criteria using weighted linear optimization. Computers and Operations Research. 33(3), 695–700 (2006)

[2] Zhou, P., Fan, L.: A note on multi-criteria ABC inventory classification using weighted linear optimization. European Journal of Operational Research. 182(3), 1488–1491 (2007)

[3] Ng, W. L.: A simple classifier for multiple criteria ABC analysis. European Journal of Operational Research. 177(1), 344–353 (2007)

[4] Hadi-Vencheh, A.: An improvement to multiple criteria ABC inventory classification. European Journal of Operational Research. 201(3), 962–965 (2010)

[5] Chen, J.X.: Peer-estimation for multiple criteria ABC inventory classification. Computers & Operations Research. 38(12), 1784–1791 (2011)

[6] Guvenir, H.A., Erel, E.: Multicriteria inventory classification using agenetic algorithm. European Journal of Operational Research. 105(1), 29–37 (1998)

[7] Tsai,C.Y.,Yeh,S.W.: A multiple objective particle swarm optimization approach for inventory classification. Int. J. Production Economics 114(2), 656–666 (2008)

[8] Mohammaditabar,D.,Ghodsypour,S.H., O'Brien,C.: Inventory control system design by integrating inventory classification and policy selection. Int. J. Production Economics. 140(2), 655–659 (2011)

[9] Patrovi,F.Y.,Anandarajan,M.: Classifying inventory using an artificial neural network approach. Computer and industrial engineering. 41(4), 389–404 (2002)

[10] Yu, M.C.: Multi-criteria ABC analysis using artificial-intelligence-based classification techniques. Expert Systems with Applications 38(4), 3416–3421 (2011)

[11] Flores, B.E, Olson, D.L, Dorai VK.: Management of multi-criteria inventory classification. Mathematical and Computer Modelling. 16(12), 71–82 (1992)

[12] Partovi, F.Y.,Burton,J.: Using the analytic hierarchy process for ABC analysis. International Journal of Operations and Production Management 13(9), 29–44 (1993)

[13] Bhattacharya, A., Sarkar, B., & Mukherjee, S. K.: Distance-based consensus method for ABC analysis. International Journal of Production Research. 45(15), 3405–3420 (2007)

[14] Roy,B.: ElectreIII;Algorithme de classement base sur une representation floue des preferences en presence de criteres multiples.CahiersdeCERO. 20(1), 3–24 (1978)

[15] Brans, J.P. , Mareschal, B., & Vincke, Ph.: PROMETHEE: a new family of outranking methods in multicriteria analysis. J.P Brans, editor,Operational Research, IFORS 84, 477–490 (1984)

[16] Saaty, T.L.: The analytic hierarchy process. McGraw-Hill International Book Co. (1980)

[17] Simos, J.: Evaluer l'impact sur l'environnement: Une approche originale par l'analyse multicritére et la négociation. Presses Polytechniques et Universitaires Romandes, Lausanne. (1990)

[18] Shannon, C. E., Weaver, W.: The mathematical theory of communication. Urbana:The University of Illinois Press. (1947)

[19] Mladenovic', N., Hansen, P.: Variable neighborhood search. Computers and Operations Research. 24(11), 1097–1100 (1997)

[20] Babai, M. Z., Ladhari, T., & Lajili, I.: On the inventory performance of multi-criteria classification methods: empirical investigation. International Journal of Production Research. 53(1), 279–290 (2015)