

A Scenario-Based Inventory Optimization Approach for a Multi-Echelon Network Considering Lateral Transshipment

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Abstract. Nowadays, Companies have to deal with an uncertain demand which is more difficult to handle when it has a non-stationary pattern. Simultaneous decrease of customer service and increase of stock costs are the most significant effects of such demand uncertainty. To deal with this issue, inventory optimization models must be adapted to cover a multi-echelon network structure and to consider alternative sourcing strategies such as lateral transshipment and multiple sourcing. In this work, a scenario-based modeling approach is proposed to solve a multi-echelon inventory optimization problem considering a non-stationary demand. The model considers minimizing the total cost that is composed of inventory holding, transportation and backordering costs. Lateral transshipment is applied to reduce inventory backorders. Another important factor in inventory optimization is flexibility. Sourcing from multiple distribution centers is a strategy to reduce supply disruption risk and to cover the possible shortage. Multiple-sourcing is considered in this research and has been analyzed in different scenarios. Several test problems are generated by Monte Carlo scenario samples and corresponding sample average approximations programs, are solved to validate the applicability of the model and to evaluate the impact of lateral transshipment and multiple sourcing.

Keywords: Multi-echelon supply chain, Inventory Optimization, Lateral Transshipment, Multiple-sourcing, Non-stationary Demand.

1 Introduction

Multi-echelon supply chains is a common network structure for large scale companies that are deployed globally and have to manage a high number of products and large market zones. However, traditional inventory optimization models tend to reduce the problem to a single echelon setting in order to keep it solvable and thus derivate optimal properties [1]. Over the last decade, multi-echelon inventory optimization models are getting more significant mostly because the current environment dynamics impose to look ahead classical sourcing and distribution strategies ([2]), and also that recent information technology advances have made it feasible to manage it in practice [3]. Early researches on the inventory optimization problem in a multi-echelon network structure indicate the advantages of considering the whole network together instead of optimizing each site/stage separately [3-5]. A framework for positioning the strategic safety stock in a multi-echelon supply chain is developed in [4] considering a base-stock policy with a common review period. They proposed a model to evaluate the inventory requirements at each stage as a function of the service times by developing an optimization algorithm for finding the service times that minimize the holding cost for the safety stock in the supply chain. Another multi-echelon and tree structure supply chain model is presented in [5], while the demand and lead time are forecasted based on a neuro-fuzzy calculation. A general stochastic model for a multi-echelon inventory optimization system based on risk inflated effecting demand is proposed in [6]. The results of the proposed model are compared to the DRP method and have already shown a significant improvement. In order to improve the performance of DRP systems, a fuzzy mathematical model which uses the interval mean value concept for the inventory optimization problem in multi-echelon supply-distribution networks [7]. It is shown that the minimum total cost under the continuous review policy can be obtained by using this method. A joint replenishment strategy is applied to the inventory system in [8] to handle a multi-product multi-echelon inventory control model. They simulated the model under three different ordering strategies to show the benefits of the proposed model in terms of the total cost reduction. A novel approach for multi period (R, Q) inventory models is introduced in [9]. A genetic solution approach is proposed to solve a pharmaceutical distribution network problem. With this in mind, the primary intent of this research is to propose a scenario-based approach to the multi-echelon inventory optimization problem under a non-stationary demand process. It also investigates to which extent lateral transshipment or

multiple sourcing distribution strategies could enhance the global performance of the supply chain network under a non-stationary demand.

Moreover, in comparison to the inventory optimization models with stationary demand in the literature, much less work has been done in the case of non-stationary demand. One of the techniques to model non-stationary demand is to break the horizon into a set of stationary phases and implement a rolling-horizon approach where the optimization should be done for each demand phase [10]. Based on the modeling approach in [4], a supply chain inventory model with a non-stationary demand process is developed by formulating a single-stage inventory model that works as a part of a multi-stage system, using service-level constraints to calculate safety stocks [11]. Every stage has a base-stock policy with a review period of one time unit. Furthermore, one should mention that inventory related costs are nowadays counted as a large proportion of supply chain costs, so it is crucial to manage them efficiently. The classic design of an inventory system is hierarchical, with transportation flows from one echelon to the next. More flexible systems also allow lateral transshipments within an echelon. Lateral transshipments are stock movement between the distribution centers in the same echelon of a supply chain network [12]. A comprehensive literature review on inventory models with lateral shipments is provided by Paterson et al [13]. In the literature, two types of pooling inventory strategies are presented. In complete pooling of inventory a transshipment location shares all of its inventory in case of transshipment request while in partial pooling a distribution center shares only a fraction of its inventory, keeping the rest to meet its own demand. A further classification of these models is based on the type of transshipment employed. In a proactive model transshipment can be effected only at fixed points in time, while transshipment can occur at any time in a reactive model which is applied in the proposed model by this paper. Even though the inventory optimization problems have been widely discussed in the literature, most of them are restricted to the single source assignment rule [14]. Under the risk of disruption, several works underlined the importance to consider more flexible sourcing/distribution strategies such as like dual or multiple sourcing [15-19]. For instance, one strategy could be to replenish the majority of items using the cheaper, but slower source, and to use the fast, but expensive source only in case of stock outs caused by the volatility of demand.

In this work, we propose a stochastic modeling approach that optimizes inventory decisions in multi-echelon supply chain network under non-stationary demand and that considers lateral transshipment and multi-sourcing strategies. It builds on stochastic programming approach [20] with the use of scenarios to shape demand uncertainty and on the Sample Average Approximation (SAA) method [21] to solve a set of equivalent deterministic problems for large samples of scenarios generated with Monte Carlo method. To the best of our knowledge, no paper has dealt with all the above mentioned aspects of inventory optimization problem. So our contributions to the literature are twofold: (1) To propose multi-echelon inventory optimization model with a scenario-based approach to handle non-stationary demand process; and (2) To introduce lateral transshipment and multiple-sourcing strategies in a multi-echelon network in order to improve its flexibility and its capabilities in reducing shortages.

The remainder of this paper is organized as follows. In Section 2 the problem context is described. In Section 3, a generic scenario-based inventory optimization model with stochastic demand is first developed and then it is extended to consider lateral transshipment and multiple sourcing features. In Section 4, computational results and managerial insights of a set of problem instances are provided.

2 Problem Definition

A common supply chain network structure consists on a set of deployed production-distribution centers (PDCs) or dedicated distribution centers (DCs) defined in a multi-echelon setting. At the tactical level of the supply chain, when a make-to-stock policy is considered, a key decision is related to the positioning of inventories in time and space. Without loss of generality, this paper considers a three-echelon supply chain including implicit suppliers, a set of production-distribution centers (PDC), a set of distribution centers (DC) and a customer zones stage (i.e. consumption points). As it is illustrated in Figure 1, each stage is fed from the upper echelon or the same echelon (lateral transshipment) and feeds the below ones. The demand from customer zones arrives to the DCs and is satisfied from the DCs on-hand inventory. DC's orders are sent to the PDCs to be provided. The PDC's needs are sourced from reliable suppliers. A tactical planning horizon (ex: yearly) is considered which is partitioned into a set of planning periods. Consumption points' demand is stochastic and non-stationary from a planning period to another, and when it could not be satisfied, it would be backordered.

To the problem description above, a multi-echelon inventory optimization approach under a periodic inventory review policy is considered. Three stochastic multi-echelon inventory optimization models are formulated. The first model is a scenario-based inventory optimization model with single sourcing. The second model enables multiple sourcing at the distribution stage to enhance flexibility and the third model considers lateral transshipment between DCs. According to periodic review, inventory level of each product is inspected at the beginning of each period and all replenishments are originated base on these reviews. Demand is received from a customer and the model decides to assign it to a DC. In the first and third models the assignment would be for all the scenarios and periods, however in second model the demand could be assigned to different DCs among all periods and scenarios. The flows between DCs and customer zones are consisted of the demand of the actual period, backordered products in previous periods and current period. When it comes to the replenishment process, an extra option as lateral transshipment is available in the third model. The DCs could receive their products via lateral transshipment which could be more expensive; however, the orders would be delivered with a shorter lead-time.

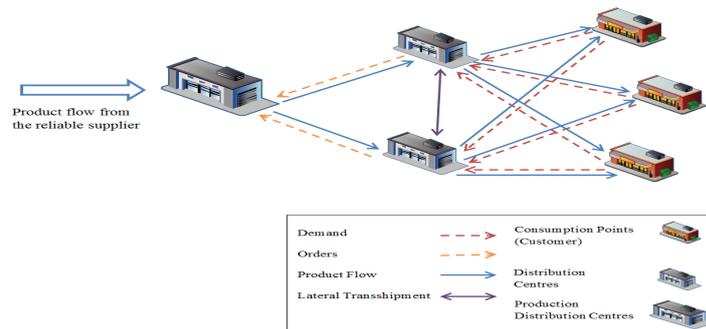


Figure 1: Multi-echelon supply chain network

One should also mention that demand planning is done on a rolling horizon basis with periodic updates and events occurring only at the start/end of a period. The lead time is a pre-planned integer number of periods covering the transportation time plus the order processing, picking, loading, reception and inspection delays. It is assumed that lead times are multiple of the review periods and the locations of the platforms are fixed. In this context, the non-stationary demand process is characterized by the following expression ([22]).

$$D_t = base + slope \times t + season \times \sin\left(\frac{2\pi}{Season\ cycle} \times t\right) \times C \times t + noise + snormal() \times t$$

Where D_t is the demand in time t , $base$ is the average demand, $season$ is a seasonal factor, $noise$ is the coefficient of demand variation, and $snormal()$ is a standard normal random number generator with the parameters mean (μ) and variance (σ^2). If C is equal to $1/t$, the demand pattern has an additive seasonality, otherwise, if C is equal to 1, it has a multiplicative seasonality. The expression in the last term of the equation provides a multiplicative noise. Using the different combinations of parameters ($base$, $slope$, $season\ cycle$, $noise$) we can generate the demand patterns. Reviewing inventory in the generated test problems is assumed to be done every week; however the demand is generated daily. Table 1 shows an example of a generic characterization of a non-stationary demand process based on a classification of customers' zones (large, medium and small). This instance will be used further in the experiments section. We notice that a Monte Carlo procedure is employed here to generate daily demands with the parameters above. With this procedure all the scenarios generated are equiprobable as it will be considered in SAA models hereafter.

Table 1: Demand process characteristics

	<i>Base</i>	<i>Slope</i>	<i>season</i>	<i>Season Cycle</i>	<i>Noise</i>	μ	σ^2
Small	300	80	50	12	100	2450	2450
Medium	500	80	50	12	100	550	550
Large	700	80	50	12	100	750	750

3 Modeling Approach

In this section, the three stochastic inventory optimization models are presented. All sets, indices, parameters, and variables used in these models formulation are listed below:

Indices:

- s : Index for suppliers $s \in S$
- w : Index for PDC platforms $w \in W$
- u : Index for DC platforms $u \in U$
- k : Index for customer zones $k \in K$
- t : Index for time periods (Periodic Review) $t \in T$
- ω : Index for the scenarios $\omega \in \Omega$

Parameters:

- $D_{k\omega}$: Demand of customer k in the beginning of period t under scenario ω
- $\eta_{nn'}$: Transportation cost unit between platform n and platform n' , $n = \{s, w, u\}$, $n' = \{w, u, k\}$
- h_n : Holding cost unit at platform n , $n = \{w, u\}$
- π_{uk} : Backorder cost unit at DC u for customer zone k
- C_{sw} : Purchasing cost unit PDC w from supplier s
- I_s : Inventory level of supplier s at the beginning of first period t
- $\tau_{nn'}$: Lead time from platform n to platform n' , $n = \{s, w, u\}$, $n' = \{w, u, k\}$
- M : A large positive number

Variables:

- $I_{n\omega}$: Inventory level in platform n at the end of period t under scenario ω , $n = \{w, u\}$
- $I_{n\omega}^+$: Inventory on hand in platform n at the end of period t under scenario ω , $n = \{w, u\}$
- $I_{ukt\omega}^-$: Backordered products in DC u from customer zone k at the end of period t under scenario ω
- $R_{nn't\omega}$: Received products in platform n' from platform n in the beginning of period t under scenario ω , $n = \{s, w, u\}$, $n' = \{w, u, k\}$
- $Z_{ukt\omega}$:
$$\begin{cases} 1 & \text{if customer } k \text{ is assigned to distribution center } u \text{ in period } t \text{ under scenario } \omega \\ 0 & \text{Otherwise} \end{cases}$$

The items shipped to a platform u by platform w at the beginning of period t are subtracted from the inventory of platform w at the beginning of period t , they are supposed to be in transit during the relevant lead time (τ_{wu}) subsequent periods and they are added to the inventory of platform u at the beginning of period $(t + \tau_{wu})$. Four kinds of costs are incurred within the network: purchasing, transportation, holding and backordering costs. The backordering and holding costs for any given platform are linear functions of inventory on-hand at the end of the period. The purchasing cost is a given parameter for the items and it is assumed that the transportation cost is a linear function of travelled distance. The related formula is $\eta_{nn'} = \alpha_{nn'}^0 + \beta_{nn'}^1 \cdot R_{nn'}$, where $\alpha_{nn'}^0$ and $\beta_{nn'}^1$ are respectively the fixed and variable transportation costs from platform n to platform n' ($n = \{s, w, u\}$, $n' = \{w, u, k\}$).

3.1 First model: Multi-Echelon Inventory Optimization Model -Single Sourcing (IO-SS)

According to the above mentioned notations, the first model is formulated as follow:

Objective Function

$$SAA(\Omega^N) = \frac{1}{N} (Min \sum_{t \in T} \sum_{\omega \in \Omega^N} (\sum_{s \in S} \sum_{w \in W} (C_{sw} + \eta_{sw}) \cdot R_{swt\omega} + \sum_{w \in W} h_{wt} \cdot I_{wt\omega}^+ + \sum_{w \in W} \sum_{u \in U} \eta_{wu} \cdot R_{wut\omega} + \sum_{u \in U} h_{ut} \cdot I_{ut\omega}^+ + \sum_{u \in U} \sum_{k \in K} (\eta_{uk} \cdot R_{ukt\omega} + \pi_{uk} \cdot I_{ukt\omega}^-))) \quad (1)$$

The objective function in (1) consists of two different parts. The first part minimizes the total related inventory cost in PDCs and purchasing and transportation costs for the flow between the suppliers and PDCs. The second part, in addition to the related inventory cost in the DCs, minimizes the backorder and transportation cost regarding to the flows between the DCs and customers.

$$I_{ut\omega}^+ = I_{u,t-1,\omega}^+ + \sum_{w \in W} R_{wut\omega} - (\sum_{k \in K} x_{ukt\omega} + \sum_{k \in K} I_{uk,t-1,\omega}^- - \sum_{k \in K} I_{ukt\omega}^-) \quad \forall u \in U, t \in T, \omega \in \Omega^N \quad (2)$$

$$I_{wt\omega}^+ = I_{w,t-1,\omega}^+ + \sum_{s \in S} R_{swt\omega} - \sum_{u \in U} R_{wu(t+\tau_{wu})\omega} - \sum_{u \in U} I_{wut\omega}^- \quad \forall w \in W, t \in T, \omega \in \Omega^N \quad (3)$$

Equations (2), (3) indicate the inventory on hand in DCs by balancing the flows-in and flows-out of in each center, period and scenario.

$$I_{ut\omega} = I_{u,t-1,\omega} + \sum_{w \in W} R_{wut\omega} - \sum_{k \in K} x_{ukt\omega} \quad \forall u \in U, t \in T, \omega \in \Omega^N \quad (4)$$

$$I_{wt\omega} = I_{w,t-1,\omega} + \sum_{s \in S} R_{swt\omega} - \sum_{u \in U} R_{wu(t+\tau_{wu})\omega} \quad \forall w \in W, t \in T, \omega \in \Omega^N \quad (5)$$

$$I_{st\omega} = I_{s,t-1,\omega} - \sum_{w \in W} R_{sw(t+\tau_{sw})\omega} \quad \forall s \in S, t \in T, \omega \in \Omega^N \quad (6)$$

$$\sum_k R_{uk(t+\tau_{uk})\omega} = \sum_{k \in K} x_{ukt\omega} + \sum_{k \in K} I_{uk,t-1,\omega}^- - \sum_{k \in K} I_{ukt\omega}^- \quad \forall u \in U, t \in T, \omega \in \Omega^N \quad (7)$$

Equations (4) - (6) point to the inventory levels of each center regarding to the inventory levels in the previous periods, outgoing flows and inflows (received the order than were made before). (7) shows the outgoing flow of DCs by taking the demand and backordered products into account.

$$D_{kt\omega} = \sum_{u \in U} x_{ukt\omega} \quad \forall k \in K, t \in T, \omega \in \Omega^N \quad (8)$$

$$\sum_{t \in T} \sum_{\omega \in \Omega^N} x_{ukt\omega} \leq M \cdot \sum_{t \in T} \sum_{\omega \in \Omega^N} Z_{ukt\omega} \quad \forall u \in U, k \in K \quad (9)$$

$$\sum_{u \in U} \sum_{\omega \in \Omega^N} \sum_{t \in T} Z_{ukt\omega} = 1 \quad \forall k \in K \quad (10)$$

$$I_{ukt\omega}^-, I_{nt\omega}^+, R_{nn't\omega} \geq 0, n = \{s, w, u\}, n' = \{w, u, k\} \\ R_{swt\omega} = 0 \quad \forall t = 1, 2, \dots, \tau_{sw}, w \in W, \omega \in \Omega^N, n = \{s, w, u\}, n' = \{w, u, k\} \\ I_{u,0,\omega}^+ = A \quad \forall u \in U, I_{w,0,\omega}^+ = B \quad \forall w \in W, I_{u,0,\omega} = A \quad \forall u \in U, I_{w,0,\omega} = B \quad \forall w \in W \quad (11)$$

Constraints (8) - (10) indicate the single sourcing process. Constraint (8) assigns the demands from each customer to the DCs. Equations (9), (10) enforce the model to assign a unique source for each customer. Feasible region for the variables are enforced by constraint (11). A and B are inventory level of the platforms in the beginning of the first period.

3.2 Second Model: Multi-Echelon Inventory Optimization Model -Multiple-Sourcing (IO-MS)

This model allows the multiple sourcing option. The demand of each customer could be assigned to different DCs. The model is somehow the same as the multi-echelon inventory optimization model. The constraint (10) is modified to come up with the multiple sourcing inventory optimization model.

Objective Function

Objective function (1)

Subject to:

$$\sum_{u \in U} \sum_{\omega \in \Omega^N} \sum_{t \in T} Z_{ukt\omega} \geq 1 \quad \forall k \in K \quad (12)$$

Constraints (2) to (9), (11)

Constraint (12) authorizes the customers to have multiple sources. The other constraints are the same as the first proposed model.

3.3 Third Model: Multi-Echelon Inventory Optimization Model with Lateral Transshipment (IO-LT)

The third model differs from the first and second models in the objective functions and constraints (2), (4), and (11). This model allows the flows between the DCs u as an option to avoid the shortage. Lateral transshipment flows are shown by the arc (u, u') .

Objective Function

$$SAA(\Omega^N) : \frac{1}{N} (Min \sum_{\omega \in \Omega^N} \sum_{t \in T} (\sum_{s \in S} \sum_{w \in W} (C_{sw} + \eta_{sw}) \cdot R_{swt\omega} + \sum_{w \in W} h_{wt} \cdot I_{wt\omega}^+ + \sum_{w \in W} \sum_{u \in U} \eta_{wu} \cdot R_{wut\omega} + \sum_{u \in U} \sum_{u' \in U \setminus \{u\}} \eta_{uu'} \cdot R_{uu't\omega} + \sum_{u \in U} h_{ut} \cdot I_{ut\omega}^+ + \sum_{u \in U} \sum_{k \in K} (\eta_{uk} \cdot R_{ukt\omega} + \pi_{uk} \cdot I_{ukt\omega}^-))) \quad (13)$$

The transportation costs for the lateral transshipment flows are added to the objective function.

$$I_{ut\omega}^+ = I_{u,t-1,\omega}^+ + \sum_{w \in W} R_{wut\omega} + \sum_{u' \in U'} R_{uu't\omega} - \sum_{u' \in U'} R_{uu'(t+\tau_{u'})\omega} - (\sum_{k \in K} x_{ukt\omega} + \sum_{k \in K} I_{uk,t-1,\omega}^- - \sum_{k \in K} I_{ukt\omega}^-) \quad \forall u \in U, t \in T, \omega \in \Omega^N \quad (14)$$

$$I_{ut\omega}^- = I_{u,t-1,\omega}^- + \sum_{w \in W} R_{wut\omega} + \sum_{u' \in U'} R_{uu't\omega} - \sum_{u' \in U'} R_{uu'(t+\tau_{u'})\omega} - \sum_{k \in K} x_{ukt\omega} \quad \forall u \in U, t \in T, \omega \in \Omega^N \quad (15)$$

Constraints (3), (6) to (11)

Equations (14), (15) are respectively equivalent to the equations (2), (4). The transshipment product flows are considered regarding to the lead time.

4 Numerical Experiment

In this section, we numerically compare the three proposed models. Table (2) indicates the generated problems setting. The model is solved for 3 different sizes and the results, in terms of objective function's cost performance, are compared. It is worth mentioning that since the whole network and parameters are exactly the same, the results are totally comparable.

Table 2: Test Problem Setting

W	U	K	π_{uk}	h_w	h_u	B	A	C_{sw}	τ_{sw}	τ_{wu}	$\tau_{uu'}$
2	2	10	50	2	5	30000	15000	4	3	2	1
2	4	30	75	5	10	60000	30000	8	3	2	1
4	4	60	100	5	10	120000	60000	8	3	2	1

Table 3 shows the numerical results for 3 different problem sizes. The problem has been set for a period of 25 weeks and 50 scenarios. Transportation, purchasing and holding cost parameters are planned based on the mentioned function in Section 3.

Table 3: Numerical Results

	W	U	K	IO-SS	IO-MS	IO-LT	Objective Function	Transportation Cost	Purchasing cost	Holding Cost	Backorder Cost	Computational time (seconds)
1	2	2	10	*			20 230 290	2 817 600	9 446 200	539 090	7 427 400	12
2	2	2	10		*		18 383 190	2 722 500	9 446 200	528 790	5 685 700	14
3	2	2	10			*	17 862 600	6 070 100	9 327 200	240 000	2 225 300	14
4	2	4	30	*			99 598 200	14 565 000	45 189 000	3 502 200	36 342 000	213
5	2	4	30		*		92 509 400	14 476 000	46 815 000	1 876 400	29 342 000	268
6	2	4	30			*	75 311 000	19 525 000	45 020 000	280 000	10 486 000	259
7	4	8	60	*			258 321 700	33 769 000	117 660 000	9 304 700	97 588 000	2736
8	4	8	60		*		226 820 300	40 381 000	105 660 000	4 427 300	76 352 000	3102
9	4	8	60			*	177 411 000	51 525 000	86 790 000	847 000	38 249 000	2910

Figure 2 compares the results for the problem numbers 4, 5 and 6. The behavior of the other problem sizes is the same. It shows the huge backorder cost reduction by using the lateral transshipment. In this example the lateral transshipment cost is 1.25 times higher than the regular transportation cost. Since multiple sourcing allows the customer to supply from different DCs or even split their demands between them, the transportation cost is optimized in the final echelon. The backorder cost is reduced by enabling the multiple sourcing. The customers do not dependent only on one DC to supply the demand, so the network is more flexible. As a result, a huge backorder and holding cost drop occurs by considering the lateral transshipment flows and multiple sourcing.

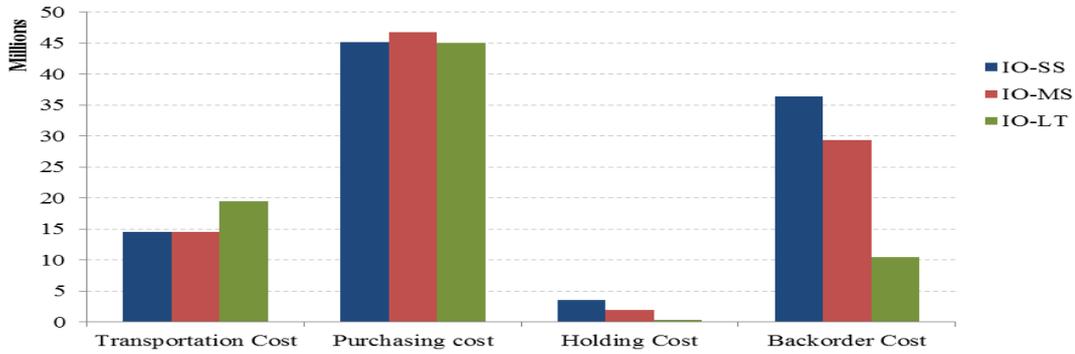


Figure 2: Comparison between the models

The lateral transshipment cost is one of the vital parameters for the decision makers. It is useful to analyze the variation of the total cost by applying different lateral transshipment costs base on the regular transportation cost. To compare the lateral transshipment with regular flow, α is defined as a parameter ($\alpha = \frac{\eta_{uu'}}{\eta_{wu}}$). Figure 3 shows the sensitivity analysis with respect to α .

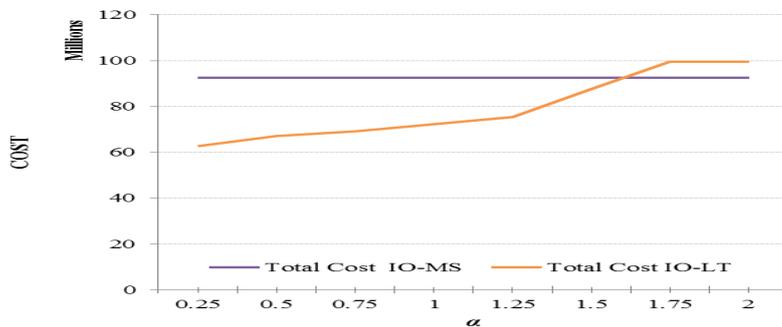


Figure 3: Sensitivity analysis of the lateral transshipment cost parameter (α)

In this case, when the lateral transshipment cost is too high (α is higher than 1.75), the model does not use lateral transshipment flows. It would work just like the first model (IO-SS), thus, multiple-sourcing is the best solution when the transportation cost between DCs is high.

5 Conclusions

This paper presents an approach to tackle a multi-echelon inventory optimization problem under non-stationary demand. Lateral transshipment has been considered in the multi-echelon inventory problem to reduce shortages. Multiple sourcing is enabled to enhance the flexibility in the network in the second model. Several test problems with different settings have been generated by using Monte Carlo techniques to validate the inventory optimization model and to evaluate the effect of lateral transshipment and multiple sourcing. The results shows a great backorder cost reduction using lateral transshipment flow; however the transportation cost is higher in comparison to the single sourcing and multiple sourcing models. According to the results, the multiple sourcing inventory optimization model is the efficient one when transportation cost between DCs is high. The sensitivity analysis on the lateral transshipment cost

shows that it could be considered as a great option when α is inferior than 1.75. It improves flexibility as well by allowing each customer to select different DCs with different costs to satisfy the demand. Further researches could consider disruption in the DCs. The multiple sourcing model would be more interesting in the cases with DC disruption. Another future research opportunities in this field could be on testing this approach with a more generic inventory policy, such as the reorder point order-up-to-level (s,S) policy.

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