A DSS approach for heterogeneous parallel machines scheduling with due windows, processor-&-sequence-dependent setup and availability constraints

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Abstract. This paper describes the optimization module of a Decision Support System (DSS) devoted to the scheduling in a fertilizer production plant, owning several parallel lines of heterogeneous characteristics, with setup time depending on sequence and lines, and taking into account non-availability constraints.

Keywords: Decision Support System, Optimization, Scheduling, Heterogeneous Parallel Processors, Sequence Dependent Setup, Non-Availability Constraints.

1 Introduction

The fertilizer plant of OCP SA is made of several heterogeneous parallel lines; the production of an order implies a cleaning operation if the produced quality differs from the previous one. Currently, the scheduling problem is defined and solved (empirically) locally; this postulates that the solution is feasible because the required inputs are available and the output storages are sufficient. It is not always the case. The proposed DSS is designed to find an optimal solution taking into account its upstream and downstream consequences. This DSS is analyzed in Giard *et al*. [41]. This paper is centered on the description of the scheduling optimization problem that takes into account simultaneously several characteristics generally studied separately. Two models are designed. The first one allows to find quickly an optimal solution respecting all local constraints and the DSS examines its upstream and downstream implications to allow, if necessary, for a reformulation of the problem. The second one adds relations taking into account, upstream and downstream constraints in its formulation; its size implies that it can be only used for small problems. Then, the DSS privileges the first model and submit a MILP problem to solve that will be modified if its optimal solution is infeasible due to upstream and downstream constraints (relaxed in model 1). Due to space constraints, the DSS is not presented here.

The literature review is presented in section 2 and the two models are described in section 3

2 Literature review

We have analyzed 40 recent articles published in the best journals, by using an analysis grid organized along five axes. This analysis is summarized in table 2 whose last line shows the characteristics retained simultaneously in our approach.

Characteristics of processors. The production system is made of parallel processors. They can be *identical*, if any job can be processed by any machine with the same production time, or *heterogeneous*, in the opposite case. The *availability* of those processors may be permanent or not (preventive maintenance…); at the beginning of the schedule, all processors may be available or not (orders in progress).

Characteristics of jobs. Jobs can be all *available* to be launched when schedule starts, or not (progressive arrivals). In both cases, the characteristics of all jobs are known. *Due dates constraints* may have to be met (through upper bounds or time windows) or not considered. *Preemption* may be authorized or not. In some papers, automatic splitting of the ordered quantity is integrated. The consequences of the solution, upstream and downstream of the supply chain, may imply the introduction of additional constraints in the problem definition

Characteristics linked simultaneously with jobs and processors. Heterogeneity implies differences of production times and the impossibility, for some processors, to treat some productions. Setup times, where applicable, may depend on the sequence and/or the processor.

Optimization criteria. Few optimization criteria use an economic point of view; most relate to efficiency (makespan..). Due to lack of space, this aspect is not considered here.

Paper's aims. A scientific paper is written with a specific aim. Three categories of targets can be identified: the *numerically-oriented* ones (new algorithm to solve a specific class of problems, analysis of the limits of existing algorithm…); the *model-oriented* ones (new formulation of a complex problem, sometimes preceded by a representative case study or followed by a small academic case study used to illustrate problem formalization); another category, not encountered in the surveyed papers, is description-oriented, devoted to explanation of complex, real situations.

3 Problem formulation

A set $\mathcal{O}$ of O orders $(o = 1..\mathrm{O})$ must be scheduled on several L non-identical parallel lines $(l = 1..\mathrm{L},\ \mathrm{L{>}O})$. The L first orders $(o = 1..\mathrm{L})$ are currently in progress at the beginning of the scheduling problem, as they were launched before; they define the set $\mathcal{O}'$. The $(\mathrm{O} - \mathrm{L})$ following orders $(o = \mathrm{L}+1..\mathrm{O})$ are the new orders to be scheduled; they define the set $\mathcal{O}''$. L fictitious orders are added, one per line $(o = \mathrm{O}{+}1..\mathrm{O}{+}\mathrm{L})$, to be the last scheduled order on each line (they act like fictitious tasks in the classic MILP formulation of the project scheduling problem); the set $\mathcal{O}'''$ $(o = \mathrm{L}{+}1..\mathrm{O}{+}\mathrm{L})$ is made of the fictitious and new orders.

Table 1. Example of definition of a set of orders (with L=2 and O=7)

		Orders to launch or fictitious order $\mathcal{O}'''$						
orders in progress or to launch $\mathcal{O}$								
orders in progress $\mathcal{O}'$		new orders to launch $\mathcal{O}''$					fictitious orders	
1	2	3	4	5	6	7	8	9

Table 2. Main characteristics of the analyzed papers

Paper# (see reference list)	Goals / Inputs section – Modeling / Problem formalization	Goals / Inputs section – Specific resolution algorithm	Goals / Inputs section – Use of general resolution methods	Characteristics of machines – Typology of parallel processors Identical (I), Heterogeneous (H)	Characteristics of machines – Availability level of parallel processors – Total at the beginning of the scheduling	Characteristics of machines – Availability level of parallel processors – Taking into account of shutdowns processors	Characteristics of jobs – Possibility of delayed arrival of some job	Characteristics of jobs – Jobs due dates – Ignored	Characteristics of jobs – Jobs due dates – Taken into account – Later date	Characteristics of jobs – Jobs due dates – Taken into account – time windows	Characteristics of jobs – Taking account of the consumption of input in the problem formulation	Characteristics of jobs – Consideration of the storage constraint of production finished	Characteristics of jobs – Possibility of job preemption	Characteristics of jobs – Taking into account the priority of job	Joint characteristics of jobs and machines – Preemption	Joint characteristics of jobs and machines – Machine dependent production time	Joint characteristics of jobs and machines – Setup time – Nil	Joint characteristics of jobs and machines – Setup time – Machine dependent	Joint characteristics of jobs and machines – Setup time – Sequence dependent	Joint characteristics of jobs and machines – Possible implementation of a job on any parallel machine
1		X		H	X	X			X							X	X			X
2		X		I	X	X			X							X		X	X	X
3		X		I		X				X									X	X
4		X		I				X						X			X			X
5	X	X		H	X	X			X					X		X	X			X
6	X	X		H	X	X		X								X		X	X	
7	X	X		H	X	X			X							X		X	X	
8	X	X		H	X				X					X		X			X	X
9		X		H	X	X			X							X			X	X
10	X	X		H	X	X			X		X					X	X			X
11	X	X		I	X	X	X			X							X			
12		X		I	X	X	X		X										X	X
13		X		I	X	X		X											X	X
14	X	X		I				X					X		X		X			X
15	X	X		I	X	X	X		X								X			X
16		X		H	X	X			X							X		X	X	X
17			X	H	X	X			X					X					X	X
18	X	X		I		X		X									X			
19	X	X		H	X				X							X				X
20	X	X		H	X	X			X							X		X	X	X
21		X		I				X									X			X
22		X		I				X							X		X			X
23		X		H	X	X			X							X		X	X	X
24	X	X		H	X	X		X								X		X	X	X
25				H																
26	X	X		H	X	X			X		X					X	X			X
27	X	X		I	X	X		X											X	X
28	X	X	X	H	X	X		X											X	
29	X	X		H		X	X		X					X		X			X	X
30	X	X		H	X	X			X					X		X		X	X	X
31		X		I	X	X	X			X				X			X			X
32		X		I	X	X	X		X										X	X
33		X		I	X	X	X		X								X			
34	X	X		H	X	X		X								X		X	X	X
35	X	X		H	X	X		X											X	X
36	X	X		H	X	X		X						X		X			X	X
37		X		I	X	X	X		X					X					X	X
38		X		I	X	X			X					X					X	X
39		X		I	X			X						X			X			X
40	X		X	H	X	X			X							X		X	X	X
41 (Us)	X		X	H	X		X			X	X	X			X	X		X	X	

In MILP, time is defined with periods (e.g. hours) and not with dates. As non-availability periods are planned in the lines (mainly due to maintenance…), the absolute period index p (p=1..P) must be replaced by the relative period index π_{lp} which depends on the line (see example in Table 3). Its use is mandatory in case of order due dates, taken into account in our models, and if there are time constraints on some inputs and/or some outputs. In absence of non-availability periods, $\pi_{lp} = p, \forall l$. Work interrupted by maintenance can be resumed without changing its total processing time.

Table 3 Example of definition of relative time

Maintenance if $\psi_{lp}=1$

ψ_{lp}		Period p (absolute time)									
		$p=1$	$p=2$	$p=3$	$p=4$	$p=5$	$p=6$	$p=7$	$p=8$	$p=9$	$p=10$
Line l	$l=1$	0	0	0	1	1	0	0	0	0	0
	$l=2$	0	0	0	0	1	1	0	0	0	0

Relative time π_{lp} (depending on absolute time and maintenance)

π_{lp}		Period p (absolute time)									
		$p=1$	$p=2$	$p=3$	$p=4$	$p=5$	$p=6$	$p=7$	$p=8$	$p=9$	$p=10$
Line l	$l=1$	1	2	3			4	5	6	7	8
	$l=2$	1	2	3	4			5	6	7	8

Two models are introduced. In both models, the orders inherit information from the references to produce: the Boolean parameter $\beta_{lj}=1$ if the output reference r required by order j (information given by table R_j) can be produced by line l; production rate ω_{lj} of order j on line l copies that of reference r on line l; setup times (and setup costs) involved by a change of output reference from one order to the following one on the same line, are added to the processing time φ_{lj} to give the production time θ_{lij} (and production cost γ_{lij}) of order j following order i on line l. This transformation simplifies the two versions of our scheduling model.

- In model 1, the binary decision variable $x_{lij}=1$ if order j $(j\in\mathcal{O}''')$ is processed on line l just after order i $(i\in\mathcal{O})$; in this formulation, time is indirectly addressed through precedence constraints in production (relation (9)). The variable x_{lij} exists only if $\beta_{li}=1$ and $\beta_{lj}=1$. Relation (1) defines the predicate $\mathcal{H}_1$ associated with the set of the decision variables x_{lij} that makes sense. This is very helpful when using Algebraic Modeling Languages (AML), like Xpress-IVE or GAMS.

$$l,i,j \,\big|\, i\in\mathcal{O}\wedge j\in\mathcal{O}'''\wedge i\neq j\wedge\beta_{li}=1\wedge\beta_{lj}=1\Rightarrow\mathcal{H}_1=True \qquad (1)$$

- In model 2, the binary decision variable $x_{lijp}=1$ if order j $(j\in\mathcal{O}''')$ is processed on line l just after order i $(i\in\mathcal{O})$, the last production period being p. Relation (2) defines the predicate $\mathcal{H}_2$ associated with the set of the decision variables x_{lijp} that makes sense. Note that it includes the due date bounds.

$$l,i,j,p \,\big|\, i\neq j\wedge i\in\mathcal{O}\wedge j\in\mathcal{O}'''\wedge\beta_{li}=1\wedge\beta_{lj}=1\wedge \mathrm{I}_{lj}\leq\pi_{lp}\leq \mathrm{S}_{lj}\Rightarrow\mathcal{H}_2=True \qquad (2)$$

A new or fictitious order j $(j\in\mathcal{O}''')$ is allocated to a unique line $(l=1..\mathrm{L})$. An order in progress at the beginning of the scheduling period $(j\in\mathcal{O}')$ cannot be followed by more than one new order. This is enforced by relations (3) for model 1 and (4) for model 2.

$$\forall j\in\mathcal{O}''',\sum_{l,i|\mathcal{H}_1=True}x_{lij}=1 \;\;;\; \forall i\in\mathcal{O}',\sum_{l,i|\mathcal{H}_1=True}x_{lij}\leq 1 \qquad (3)$$

$$\forall i\in\mathcal{O}''',\sum_{l,j,p|\mathcal{H}_2=True}x_{lij\pi_{lp}}=1 \;\;;\; \forall i\in\mathcal{O}',\sum_{l,j,p|\mathcal{H}_2=True}x_{lij\pi_{lp}}\leq 1 \qquad (4)$$

The last period of production of order j is its delivery date y_j. This date is bounded by lower and upper dates (L_{lj} and U_{lj}) that are defined with the relative calendar of line l.

These constraints are enforced for the new orders $(\mathscr{E}_{O''})$ in model 1 by relation (5) and, in model 2, by relation (2) which restrains the scope of variable x_{lijp}. Note that the delivery dates of the orders in progress are already known $(\forall j \in \mathscr{E}_{O'}, y_j = \mathrm{L}_{jj} = \mathrm{U}_{jj})$.

$$\forall j \in \mathscr{E}_{O''}, \sum_{l,i,j | \mathscr{K}_1 = True} \mathrm{L}_{lj} \cdot x_{lij} \le y_j \le \sum_{l,i,j | \mathscr{K}_1 = True} \mathrm{U}_{lj} \cdot x_{lij} \quad (5)$$

A new order *j* produced on line *l* must have a predecessor *i* (in progress or new) produced on the same line. Relation (6) for model 1 and (7) for model 2 enforce order *i* to be produced on line *l* if $x_{lij} = 1$ (model 1) or $\sum_p x_{lij\pi_{lp}} = 1$ (model 2).

$$\forall j \in \mathscr{E}_{O''} \wedge \forall l \big| \beta_{lj} = 1, \sum_{k \in \mathscr{E}_O | k \ne j \wedge \beta_{lk} = 1} x_{lkj} = \sum_{h \in \mathscr{E}_{O'''} | h \ne j \wedge \beta_{lh} = 1} x_{ljh} \quad (6)$$

$$\forall j \in \mathscr{E}_{O''} \wedge \forall l \big| \beta_{lj} = 1, \sum_{k \in \mathscr{E}_O | k \ne j \wedge \beta_{lk} = 1} \sum_{p | \mathrm{L}_{lj} \le \pi_{lp} \le \mathrm{U}_{lj}} x_{lkj\pi_{lp}} = \sum_{h \in \mathscr{E}_{O'''} | h \ne j \wedge \beta_{lh} = 1} \sum_{p | \mathrm{L}_{lj} \le \pi_{lp} \le \mathrm{U}_{lj}} x_{ljh\pi_{lp}} \quad (7)$$

Relations (8) for model 1 and (9) for model 2 prevent order *j* to be produced as long as the production of order *i* is in progress, when both orders are produced on the same line. In these relations, the number P of periods plays the role of the "big M" constant.

$$\forall i \in \mathscr{E}_O, j \in \mathscr{E}_{O''} \big| i \ne j,\ y_j - y_i \ge \sum_{l | \mathscr{K}_1 = True} \theta_{lij} \cdot x_{lij} - \mathrm{P} \cdot (1 - \sum_{l | \mathscr{K}_1 = True} x_{lij}) \quad (8)$$

$$\forall i \in \mathscr{E}_O, j \in \mathscr{E}_{O''} \big| i \ne j, y_j - y_i \ge \sum_{l,p | \mathscr{K}_2 = True} \theta_{lij} \cdot x_{lij\pi_{lp}} - \mathrm{P} \cdot (1 - \sum_{l,p | \mathscr{K}_2 = True} x_{lij\pi_{lp}}) \quad (9)$$

Relations (10) to (12) are specific to model 2. Relation (10) links variables y_j and $x_{lij\pi_{lp}}$ in model 2. Relation (11) defines the total consumption C_p during period *p* of the considered input; it is a linear expression of the decision variables and uses the consumption rate q_{lj} of order *j* on line *l* (null if $\Psi_{lp} = 1$, see table 3). C_p is calculated as the sum of the consumption of orders in progress and of new orders. Its use will be seen below.

$$\forall j \in \mathscr{E}_{O''}, y_j = \sum_{l,i,p | \mathscr{K}_2 = True} \pi_{lp} \cdot x_{lij\pi_{lp}} \quad (10)$$

$$\forall p, C_p = \sum_{l,j \in \mathscr{E}_{O'} | l = j \wedge \Psi_{lp} = 0 \wedge \pi_{lp} \le \mathrm{U}_j} \mathrm{q}_{lj} + \sum_{l,j \in \mathscr{E}_{O''} | \beta_{lj} = 1 \wedge \Psi_{lp} = 0 \wedge \pi_{lj} \le \mathrm{U}_j} \mathrm{q}_{lj} \cdot \sum_{t \ge p, i \in \mathscr{E}_O | \beta_{li} = 1 \wedge i \ne j \wedge \mathrm{L}_j \le \pi_{lt} < \pi_{lp} + \varphi_{lj}} x_{lij\pi_{lt}} \quad (11)$$

Order *j* relates to quality output *r* (given by R_j); several orders may relate to a same quality *r*. With production ratio p_{lj} on line *l*, the total production P_{rp}, during period *p*, is given by relation (12) which is a linear expression of the decision variables.

$$\forall p, r, P_{rp} = \sum_{l,j \in \mathscr{E}_{O'} | l = j \wedge \mathrm{R}_j = r \wedge \Psi_{lp} = 0 \wedge \pi_{lp} \le \mathrm{U}_j} \mathrm{p}_{lj} + \sum_{l,j \in \mathscr{E}_{O''} | \beta_{lj} = 1 \wedge \mathrm{R}_j = r \wedge \Psi_{lp} = 0 \wedge \pi_{lj} \le \mathrm{U}_j} \mathrm{p}_{lj} \cdot \sum_{t \ge p, i \in \mathscr{E}_O | \beta_{li} = 1 \wedge i \ne j \wedge \mathrm{L}_j \le \pi_{lt} < \pi_{lp} + \varphi_{lj}} x_{lij\pi_{lt}} \quad (12)$$

The variables C_p and P_{rp} may be used in constraints added in model 2 to take into account a possible stockout of the input (depending on its availability) and/or a possible saturation of the stock where the production of quality *r* is sent (see [13]). These problems may be shown by exploring the consequences of the optimal solution of model 1, unable to take into account the physical consequences of a schedule, upstream and downstream of the fertilizer plant.

The schedule cost is given by relation (13) for model 1 and (14) for model 2, where γ_{lij} is the cost of producing order *j* on line *l*, after order *i*. These costs does not take into account expenses which must be supported whatever the decision taken.

$$Cost_1 = \sum_{l,i,j|\mathscr{K}_1=True} \gamma_{lij} \cdot x_{lij} \tag{13}$$

$$Cost_1 = \sum_{l,i,j,p|\mathscr{K}_2=True} \gamma_{aij} \cdot x_{lij\pi_{lt}} \tag{14}$$

Several schedules may have the same minimum value of $Cost_1$. Among them, schedules with the earliest delivery dates are usually preferred. This is obtained when using $Cost_2$ given by relation (15) for model 1 and (16) for model 2.

$$Cost_2 = Cost_1 + 0.01 \cdot \sum_{j\in C''} (y_j - \sum_{l,i|\mathscr{K}_1=True} L_{lj} \cdot x_{lij}) \tag{15}$$

$$Cost_2 = Cost_1 + 0.01 \cdot \sum_{j\in C''} (y_j - \sum_{l,i,j,p|\mathscr{K}_2=True} L_{lj} \cdot x_{lij\pi_{lt}}) \tag{16}$$

Let us illustrate model 1 (for an illustration of model 2, see [41]). As in table 1, the scheduling problem deals with 2 lines (with the maintenance program of table 2) and 7 orders. These orders relate to 3 output references (R_j). Some of these references cannot be produced on all the lines. Tables 4 give for each order *j* and each line *l*, the upper and lower bounds of the due dates (L_{lj} and U_{lj}), the production rates p_{lj} of the order, the consumption rate ω_{lj} of the critical input, the processing times φ_{lj}; the setup time is assumed to be 2 hours, whatever the sequence $i \to j$ $(i \neq j)$ and whatever the line *l* where *j* is produced. The cost function is $\gamma_{lij} = 10 \cdot \varphi_{lj} + 10 \cdot \beta_{li} \cdot \beta_{lj}$.

Tables 4: data of the scheduling problem

φ_{lj}	$j=1$	$j=2$	$j=3$	$j=4$	$j=5$	$j=6$	$j=7$
Line $l=1$	5	0	12	8	0	6	9
Line $l=2$	0	7	16	0	10	8	12

ω_{lj}	$j=1$	$j=2$	$j=3$	$j=4$	$j=5$	$j=6$	$j=7$
Line $l=1$	100	0	160	100	0	160	160
Line $l=2$	0	80	120	0	80	120	120

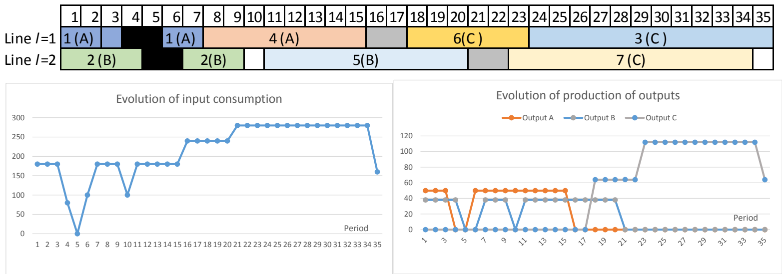
p_{lj}	$j=1$	$j=2$	$j=3$	$j=4$	$j=5$	$j=6$	$j=7$
Line $l=1$	50	0	64	50	0	64	64
Line $l=2$	0	38	48	0	38	48	48

R_j (output)	A	B	C	A	B	C	C

L_{lj}	$j=1$	$j=2$	$j=3$	$j=4$	$j=5$	$j=6$	$j=7$	$j=8$	$j=9$
Line $l=1$	7	0	10	15	0	20	30	1	0
Line $l=2$	0	9	10	0	20	20	30	0	1

U_{lj}	$j=1$	$j=2$	$j=3$	$j=4$	$j=5$	$j=6$	$j=7$	$j=8$	$j=9$
Line $l=1$	7	0	40	40	0	40	40	40	0
Line $l=2$	0	9	40	0	40	40	40	0	40

Using criterion (15), optimal solution (illustrated by the Gantt below) is $x_{1,1,4} = x_{1,4,6} = x_{1,6,3} = x_{1,3,8} = x_{2,2,5} = x_{2,5,7} = x_{2,7,9} = 1$ (the other decision variables being nil), $Cost_2 = 574.32$ and $Cost_1 = 574$. Order 5 cannot start before period 11, which induces a “hole” in the Gantt given in figure 1.



Figures 1: Optimal solution of model 1

4 Conclusion

A prototype of Decision Support System is currently experimented. The model 2 is used only for problems of small size, due to the computations it involves. The solution given for model 1 is completed by the analysis of its downstream and upstream implications to verify its feasibility and, if necessary, to redesign the problem.

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