Location-Inventory Problem with stochastic demand and lead time

Mehdi Amiri-Aref [1], Walid Klibi[1,2], Zied Babai [1]

[1] Operations Management and Information Systems Department, KEDGE Business School, France
[2] Interuniversity Research Centre on Enterprise Networks, Logistics and Transportation (CIRRELT), Canada
{mehdi.amiri-aref, walid.klibi, mohamed-zied.babai}@kedgebs.com

Abstract. In this paper, a multi-period location-inventory optimization problem characterized by uncertain demand and supply lead-time is studied. A stochastic two-stage mathematical model maximizing the total expected supply network profit is proposed. The main purpose of this work is to introduce a generic modelling approach to integrate key features of the inventory planning decisions with location-allocation design decisions with uncertain demand and lead-time. To solve the problem, a sample average approximation approach is used. The results show the strong interconnectivity between inventory decisions and strategic location-allocation decisions in a supply chain. Based on an illustrative case, it is found out that solving this problem is very difficult and time consuming and that varying inventory costs result in an important impact on the supply chain network design.

Keywords: Location-inventory problem, stochastic lead time, stochastic demand.

1 Introduction

The Supply Chain (SC) strategy of a given company involves leveraging its core competencies to achieve the ultimate goal of sustainable value creation. At the strategic level, this implies decisions on how to deploy the SC resources over a long term horizon, by locating a set of facilities, determining their capacities and mission (i.e. determine which product families are supplied, stocked and distributed by each facility) in order to maximize the economic value added of the company. Therefore, SC managers should design a Supply Chain Network (SCN) that is efficient regarding its logistic operations between the different entities involved (i.e. owned resources, partners and subcontractors), and that is robust regarding any plausible future environment that could occur (i.e. markets volatility, factors variability and trends, network disruptions,…). In the same time, the SC strategy implies decisions on the Planning and Control (P&C) system such as demand forecasting methods, inventory policies, production (MRP) and distribution (DRP) planning systems, etc., in order to manage efficiently the SC for cost saving and service maximization. However, these decisions are always taken independently from the SC design decisions: in a reengineering process beforehand or afterward, but not in an integrated way. Nevertheless, it is worthy to notice that the capability of these operational activities is dependent on the SC network structure deployed. More specifically, the deployment of facilities over the territory and the strategic deployment of stocks within these facilities are interrelated problems and must be considered together. In this way of thinking, this research work, aims at investigating some inventory considerations within the SC network design problem. It attempts to provide a methodology to integrate key features of the inventory problem at the strategic level and to discuss the complexity and solvability issues related to this integration. Note that this research work is cast in a make-to-stock business context.
The first research work that deals with integrated supply networks refers to the classical location-allocation models which are generally based on a predetermined supply structure [1]. They deal with the location of facilities in some given geographical area. Several functional expansions such as the capacity requirements, technology selection, number of sources and objective function have been proposed in the last two decades to the basic version of the facility location problem. The allocation decisions correspond to the specification of the facilities mission in term of the downstream locations and/or customers to supply. On the other hand, the aim of the inventory problem is to develop policies for quantities to be stored or produced or to be ordered in order to meet the customer demand at the operational level. New methods to calculate the optimal parameters of an inventory system with stochastic demand modelled by compound distributions are studied [2]. Tradeoff the costs of several decisions neglecting the inventory-dependent cost have also been studied under deterministic characteristics. Recently, increasing attentions are paid to designing SCNs integrated with inventory operations under uncertainty that has shaped several stochastic versions of the location-allocation problem [3]. Although major efforts have been devoted to

the development of location models with inventory management decisions, integrated models of the location-inventory problem with a static setting, i.e. where no hierarchy is established between these decisions have been proposed [4], [5]. These models assume that demand is stationary and its distribution parameters (i.e. mean and variance) are known with certainty which justifies the assumption of the use of the EOQ inventory policy (r,Q). In what follows, SCN design models considering inventory and location-inventory models under uncertainty developed in the last few years are exposed. Daskin et al. [1] developed a mixed integer nonlinear programming (MINLP) model to design location-allocation decisions within a static SCN and applied lagrangian relaxation method to solve the model. Incorporated (r,Q) inventory policy in the model, they optimized inventory level and safety stock decision variables. A stochastic location-allocation model integrated with inventory and service level with known lead-time has been developed in [7] and [8]. Shen and Qi [9] designed stochastic location-allocation-inventory problem under (r,Q) replenishment policy and known safety stock level at warehouses that motivated many researchers for extension. Ozsen et al. [10] extended the same problem incorporating transportation decision variables and applied Lagrangian relaxation method to solve the problem. Shu [11] developed a recursive solution procedure and a hybrid of GA with column generation solution method. Ozsen et al. [6] and Park et al. [12] developed a static capacitated location-allocation-inventory problem as an MINLP model optimizing safety level and order quantity under (r,Q) inventory control policy. Atamtürk et al. [13] have proposed a stochastic version of the previous models where they have focused on solving the obtained stochastic conic model. Liu et al [14] considered stochastic location-allocation strategic decisions and inventory-transportation tactical decisions in SCND problem regardless the network design hierarchy levels.

To the best of our knowledge, the stochastic customer demand and stochastic supply lead time has not been addressed in the literature that deals with the location-inventory problem. This constitutes the objective of this paper, where a multi-period location-inventory optimization problem characterized by uncertain demand and lead-time is studied.

2 Problem Description and Formulation

A typical potential distribution network can be represented by a directed graph such as the one illustrated in Figure 1. The nodes of this graph correspond to existing and potential supply sources, distribution centres (DCs) and demand zones. Within the network facilities, we assume that inventory levels of products are maintained in time: they are kept stored, in respect to the storage capacity space, before being shipped in respect to the throughput capacity and that monitored periodically. Whenever the inventory level reaches or drops below the reorder point, an order issues to sources. The simple directed arcs are associated to the transportation lanes that could be used to move products from sources to DCs, and from DCs to demand zones. The dashed directed arcs identify the information flow through the network that could be shared. Let us describe the sequence of events related to inventory control considered in this paper. After receiving demands from all market zones at the end period t, inventory position is checked and an order is issued if the inventory position is below the reorder point. The order quantity will be received at the beginning of period $t + L + 1$ where L is the lead time. Excess demand is backordered. Inventory control and planning in this study covers the inventory policy and throughput capacity settings. The former is influenced by the periodic demand and inventory-dependent costs (cost of inventory holding, backordering, and ordering) and has an impact on operating periodic decisions, while the latter is anticipating by the annual market demand affecting both strategic and operating decisions for whole the planning horizon. The SC design problem aim to select the best subset of DCs to deploy that maximizes the profit of the company. Since the decision variables mentioned above raise from different stages of decision making hierarchy, we formulate a two-stage mathematical model to keep the hierarchically interconnection of location-allocation decisions with inventory decisions.

In this problem, it is supposed that location-allocation decisions refer to select a subset of potential DC locations that could be allocated to set of supply sources. In the distribution network considered, set of demand zones are allocated to open DCs at tactical level. In other words, customers cannot be supplied from only sources. It is also supposed that inventory decisions refer to specify the best inventory level at the opened DCs during the planning horizon, to make order at right quantity and right time in order to keep a high customer service level, and to control and recovery unmet demand. The objective function here corresponds to maximizing the marginal profit earned from sales revenues minus total costs. Considering that demand is uncertain along the planning horizon, this latter is shaped by a set of demand scenarios characterising the possible needs per demand zone. Moreover, a multi-period setting is proposed with set of planning periods in order to mimic the demand process precisely and to integrate the inventory policy more accurately.

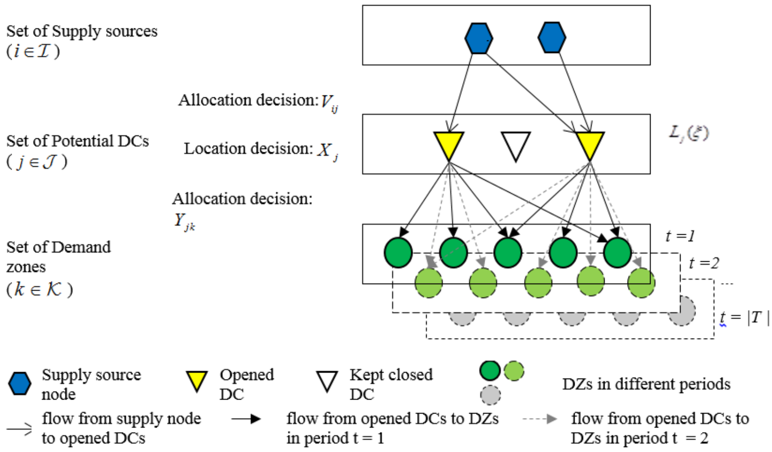

Figure 1: SC network

From the business context point of view, this study could match with the companies in which a set product from one kind of family product (considered as a single product) is purchased from a number of supply sources and is dispatched to a large number of DZs in order to maximize the marginal profit. Making a decision to open a new DC charges the company the fixed opening cost and variable operating costs which are associated with inventory-dependent costs (cost of inventory holding, backordering, and ordering). Opening decision is made at the strategic level which is applied for the whole planning horizon while operating decisions of a triggered DC are made at the tactical level depending on the periodic inventory control and replenishment policy defined at the time of opening decisions. Due to the temporal hierarchy between opening and operating decisions, a vertical integration of decisions in SCN design should be considered. In other words, neglecting the interrelationship between strategic opening and tactical operating decisions narrows the marginal profit of company and may result in increasing unsatisfied customers. In addition, unpredictable customers' demand is another issue that has significant influence on the SC effectiveness. On the other hand, opening a new DC makes the company more responsiveness to underserved DZs and reduces the transportation costs that may increase the total revenue. Thus a trade-off between the costs imposed and revenue earned can be investigated by an SMLIP.

2.1 Model notations

Here the set of indices and input parameters applied in this problem are introduced.

$i \in \mathcal{I} = \{1,\ldots,\lvert I\rvert\}$	Set of supply sources
$j \in \mathcal{J} = \{1,\ldots,\lvert J\rvert\}$	Set of potential DC locations
$k \in \mathcal{K} = \{1,\ldots,\lvert K\rvert\}$	Set of demand zones
$t \in \mathcal{T} = \{1,\ldots,\lvert T\rvert\}$	Planning horizon
$(\omega,\xi) \in \Omega$	Set of demand and lead time scenarios
f_j	Fixed cost of opening a new DC on node *j*
g_{ij}	Fixed cost of contract with transporter from source *i* to DC *j*
l_{jk}	Fixed cost of contract with transporter from DC *j* to demand zone *k*
α_{ij}	linear transportation cost in terms of distance from source *i* to DC *j*
m_{ij}	Distance from source *i* to DC *j*

β_{jk}	Linear transportation cost in terms of distance from DC *j* to demand zone *k*
n_{jk}	Distance from DC *j* to demand zone *k*
$d_{kt\omega}$	Demand of zone *k* at period *t* under scenario ω
h	Inventory holding cost per unit per period
γ	Backorder cost for unfulfilment per unit per period
ρ_k	Sales price (i.e., revenue from) satisfying demand zone *k*
δ	Material purchase cost
a	Order cost of DC located in node *j*
Cap_j	Capacity of DC *j*
r_{jt}	Reorder point of DC *j* at period *t*
ψ_i	Supply capacity of supplier *i*
M	A positive large number
π_ω	Probability of scenario ω occurrence
$L_j(\xi)$	Lead-time of DC *j* under scenario ξ

Note that the input parameters of stochastic demand and lead time scenario generation follows the Monte Carlo procedure which is represented further in the paper. Some preliminaries regarding scenario generations are described in the Appendix. The decision variables optimizing within the model are in the following:

X_j	is equal to 1 if a warehouse locates on node *j*, otherwise 0
V_{ij}	is equal to 1 if source *i* has shipment to DC *j*, otherwise 0
Y_{jk}	is equal to 1 if DC *j* has shipment to demand zone *k*, otherwise 0
$I_{jt}^{P}(\omega)$	Inventory stock position at DC *j* at period *t* under scenario ω
$I_{jt}^{+}(\omega)$	On hand inventory level at DC *j* at period *t* under scenario ω
$\overline{I}_{jt}(\omega)$	Average inventory level at DC *j* over period *t* under scenario ω
$Z_{jt}(\omega)$	is equal to 1 if a DC *j* place on order at period *t* under scenario ω , otherwise 0
$Q_{jt}(\omega)$	Amount of quantity ordered at period *t* under scenario ω from DC *j*
$W_{ijt}(\omega)$	Amount of shipment from source *i* to a DC *j* at period *t* under scenario ω
$F_{jkt}^{s}(\omega)$	Flow from DC *j* to demand zone *k* at period *t* under scenario ω that is shipped due to the available stock at DC *j*
$F_{jkt}^{n}(\omega)$	Flow from DC *j* to demand of zone *k* at period *t* under scenario ω that is not shipped due to out of stock at DC *j*
$q(\omega)$	Profit function under scenario ω

2.2 Stochastic two-stage location-inventory model

The SMLIP is a hierarchical decision problem because of the temporal hierarchy that exists between the location-allocation decisions and the inventory control and planning decisions. The SMLIP is formulated as a two-stage mathematical model in the following, in which the location-allocation decisions are optimized in the first stage and the inventory control and planning decisions are optimized in the second stage. In the presented model above, the first stage program contains objective function (1) and system constraints (2)-(5), while the second stage program contains objective function (6.1) to (6.5) and system constraints (7)-(16), with binary decision variables (17) and non-negative decision variables (18).

$$\max \sum_{\omega \in \Omega} \pi(\omega) q(\omega) - \sum_{j \in \mathcal{J}} f_j X_j - \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} g_{ij} V_{ij} - \sum_{j \in \mathcal{J}} \sum_{k \in \mathcal{K}} l_{jk} Y_{jk} \tag{1}$$

$$V_{ij} \le X_j \qquad i \in \mathcal{I}, j \in \mathcal{J} \tag{2}$$

$$\sum_{i\in\mathcal{I}} V_{ij} \geq X_j \qquad j\in\mathcal{J}, k\in\mathcal{K} \qquad (3)$$

$$Y_{jk} \leq X_j \qquad j\in\mathcal{J}, k\in\mathcal{K} \qquad (4)$$

$$\sum_{j\in\mathcal{J}} Y_{jk} \geq p \qquad k\in\mathcal{K} \qquad (5)$$

where:

$$q(\omega) = \max \sum_{j\in\mathcal{J}}\sum_{k\in\mathcal{K}}\sum_{t\in\mathcal{T}} \rho F^s_{jkt}(\omega) \qquad (\omega,\xi)\in\Omega \qquad (6.1)$$

$$-\sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{J}}\sum_{t\in\mathcal{T}} \delta W_{ijt}(\omega) \qquad (6.2)$$

$$-\sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{J}}\sum_{t\in\mathcal{T}} \alpha_{ij} m_{ij} W_{ijt}(\omega) - \sum_{j\in\mathcal{J}}\sum_{k\in\mathcal{K}}\sum_{t\in\mathcal{T}} \beta_{jk} n_{jk} F^s_{jkt}(\omega) \qquad (6.3)$$

$$-\sum_{j\in\mathcal{J}}\sum_{t\in\mathcal{T}} \left(h\bar{I}_{jt}(\omega) + aZ_{jt}(\omega)\right) \qquad (6.4)$$

$$-\sum_{j\in\mathcal{J}}\sum_{k\in\mathcal{K}}\sum_{t\in\mathcal{T}} \gamma F^n_{jkt}(\omega) \qquad (6.5)$$

s.t:

$$\bar{I}_{jt}(\omega) = \left(I^+_{jt}(\omega) + I^+_{j,t-1}(\omega) + Q_{j,t-(L_j(\xi)+1)}(\omega)\right)\Big/2 \qquad j\in\mathcal{J}, t\in\mathcal{T}, (\omega,\xi)\in\Omega \qquad (7)$$

$$I^+_{jt}(\omega) = I^+_{j,t-1}(\omega) + Q_{j,t-(L_j(\xi)+1)}(\omega) - \sum_{k\in\mathcal{K}} F^s_{jkt}(\omega) \qquad j\in\mathcal{J}, t\in\mathcal{T}, (\omega,\xi)\in\Omega \qquad (8)$$

$$I^P_{jt}(\omega) = I^+_{jt}(\omega) + \sum_{l=t-L_j(\xi)}^{t-1} Q_{jl}(\omega) - \sum_{k\in\mathcal{K}} F^n_{jkt}(\omega) \qquad j\in\mathcal{J}, t\in\mathcal{T}, (\omega,\xi)\in\Omega \qquad (9)$$

$$I^P_{jt}(\omega) \leq r_{jt} + M.\left(1 - Z_{jt}(\omega)\right) \qquad j\in\mathcal{J}, t\in\mathcal{T}, (\omega,\xi)\in\Omega \qquad (10)$$

$$\sum_{k\in\mathcal{K}} F^s_{jkt}(\omega) \leq Cap_j X_j \qquad j\in\mathcal{J}, t\in\mathcal{T}, (\omega,\xi)\in\Omega \qquad (11)$$

$$\sum_{j\in\mathcal{J}} F^s_{jkt}(\omega) = d_{kt}(\omega) - U_{kt}(\omega) + U_{k,t-1}(\omega) \qquad k\in\mathcal{K}, t\in\mathcal{T}, (\omega,\xi)\in\Omega \qquad (12)$$

$$F^e_{jkt}(\omega) \leq M.Y_{jk} \qquad e\in\{s,n\}, j\in\Im, k\in\mathcal{K}, t\in\mathcal{T}, (\omega,\xi)\in\Omega \qquad (13)$$

$$W_{ijt}(\omega) \leq \psi_i.V_{ij} \qquad i\in\mathcal{I}, j\in\mathcal{J}, t\in\mathcal{T}, (\omega,\xi)\in\Omega \qquad (14)$$

$$Z_{jt}(\omega) \leq X_j \qquad j\in\mathcal{J}, t\in\mathcal{T}, (\omega,\xi)\in\Omega \qquad (15)$$

$$Q_{jt}(\omega) \leq M.Z_{jt}(\omega) \qquad j\in\mathcal{J}, t\in\mathcal{T}, (\omega,\xi)\in\Omega \qquad (16)$$

$$Q_{jt}(\omega) = \sum_{i\in\mathcal{I}} W_{ijt}(\omega) \qquad j\in\mathcal{J}, t\in\mathcal{T}, (\omega,\xi)\in\Omega \qquad (17)$$

$$\sum_{j\in\mathcal{J}} F^n_{jkt}(\omega) = U_{kt}(\omega) \qquad k\in\mathcal{K}, t\in\mathcal{T}, (\omega,\xi)\in\Omega \qquad (18)$$

$$X_j, V_{ij}, Y_{jk}, Z_{jt}(\omega) \in \{0,1\} \qquad i\in\mathcal{I}, j\in\mathcal{J}, t\in\mathcal{T}, (\omega,\xi)\in\Omega \qquad (19)$$

$$I^P_{jt}(\omega), I^+_{jt}(\omega), Q_{jt}(\omega), W_{ijt}(\omega), F^e_{jkt}(\omega), U_{kt}(\omega) \geq 0 \qquad e\in\{s,n\}, i\in\mathcal{I}, j\in\Im, k\in\mathcal{K}, t\in\mathcal{T}, (\omega,\xi)\in\Omega \qquad (20)$$

The first term of the objective function (1) states the expected net revenues optimization from the second stage program. The other terms included in the objective function (1) contain the fixed costs composed of two terms, the fixed DC opening cost at potential locations and the transportation provider allocation costs. Constraints (2) guarantee that supply flow from sources is available for the opened DC facilities. Constraints (3) state that each opened DC can be supplied by at least one source. Constraints (4) check that demand zone supply is available from opened DCs. Constraints (5) guarantee that each DZ can be served by more than *p* facilities.

The second stage program maximizes a scenario-based objective function comprising total revenues obtained by satisfying the products to demand zones (6.1), total purchased cost from the sources (6.2), total (outbound and inbound) transportation cost in terms of distance from supplier (DC) to DC (demand zones) incurred in the network (6.3), and total inventory-dependent costs including average inventory holding costs, ordering cost (6.4), and backordering cost (6.5), all on a specific time-segment basis over the planning horizon under different scenarios. The average inventory stock (constraints (7)) that is being kept over period, $\overline{I}_{jt}(\omega)$, is the average of initial inventory at the beginning of period *t*, $I^{+}_{j,t-1}(\omega) + Q_{j,t-(L_j(\xi)+1)}(\omega)$, and the final inventory at the end of the period, $I^{+}_{jt}(\omega)$. Constraints (8) check the equilibrium inventory state transition from one period to the next period. Constraints (9) represents the inventory stock position which is the inventory on hand and inventory pipeline minus the backorder of a DC. Constraints (10) control the periodic replenishment policy in which an order from a DC is placed whenever the inventory stock position reviewed periodically drops down the reorder point. Constraints (11) declare the inventory level at each period of time cannot exceed the warehouse throughput capacity. Constraints (12) represent that the product flow from all DCs to a given DZ should consider the current demand and the possible cumulative unshipped products from DCs assigned from previous periods. Constraints (13) and (14) specify that the flow from DCs (sources) to DZs (DCs) is dependent on the allocation variables defined in the first stage. Constraints (15) and (16) guarantee that the orders are issued from open DCs and the order quantity has a value when an order is placed. Constraints (17) indicate that what is ordered should be supplied by the available sources. Note that the unshipped products causing backorder to a DZ from assigned DCs defines the unmet demand of that zone as follows: $\sum_{j\in\mathcal{J}} F^{n}_{jk\tau}(\omega) = U_{k\tau}(\omega)$ which are represented in constraint (18). Network decision variables are introduced in (19) and (20).

2.3 Sample average approximation model

Let us define the *n*-dimensional vector random variable $\mathbf{C} = \left\{C^{1}_{kt}, C^{2}_{kt}, \ldots, C^{n}_{kt}\right\}$ as the theoretical compound demand random variable of demand zone $k \in \mathcal{K}$ in period $t \in \mathcal{T}$ following the distribution function $g_1(\bullet;\boldsymbol{\kappa})$ with parameters $\boldsymbol{\kappa}$. Let's μ^{C}_{kt} and σ^{C}_{kt} be the compound mean and standard deviation demand of zone $k \in \mathcal{K}$ in period $t \in \mathcal{T}$, respectively. Also assume vector random variable **D** as the stochastic lead time random variable of DC *j* that follows distribution function $g_2(\bullet;\boldsymbol{\Lambda})$ with parameters $\boldsymbol{\Lambda}$. Both demand and lead time scenarios are generated following the Monte-Carlo procedure shown in Figure 2. $G^{-1}_1(\bullet)$ and $G^{-1}_2(\bullet)$ are the inverse of the abovementioned distribution functions.

Input: $G^{-1}_1(\bullet)$, $G^{-1}_2(\bullet)$, $k \in \mathcal{K}$, , $t \in \mathcal{T}$, $(\omega,\xi) \in \Omega$

For all $\omega \in \Omega^{N}$

  For all $k \in \mathcal{K}$ and $\tau_t \in \Phi$, $t \in \mathcal{T}$

    Generate the Uniform [0,1] random numbers rnd_{g1} and rnd_{g2}

    Compute the inverse of the compound distributions $G^{-1}_1\left(rnd_{g1}\right)$

    Compute the inverse of the distributions $G^{-1}_2\left(rnd_{g2}\right)$

  Next

Next

Figure 2. Compound demand and lead time scenario generation procedure.

Applying the Monte-Carlo procedure and generating *N* sample of compound demand and lead time scenarios, we then constructed and solved the sample average approximation (SAA) model. Based on the sample average approximation (SAA) technique [15], this problem could be reformulated for a sample of equiprobable scenarios of size *N* ($(\omega,\xi) \in \Omega$) generated by a Monte-Carlo procedure as: (2)-(5), (7)-(20), and (21). Obviously the solution quality found with the SAA model improves by increasing the scenario sample size *N*; however, it rises its complexity that makes a major issue of solvability. Considering that, the commercial optimization software CPLEX consumes a long computational time to solve the model for larger sizes due to the high number of constraints and binary decision variables.

$$\max \sum_{\omega\in\Omega^N} \frac{1}{N}\left(\sum_{j\in\mathcal{J}}\sum_{k\in\mathcal{K}}\sum_{t\in\mathcal{T}} \rho F^s_{jkt}(\omega) - \sum_{j\in\mathcal{J}}\sum_{t\in\mathcal{T}}\left(\sum_{i\in\mathcal{I}} \alpha_{ij} m_{ij} W_{ijt}(\omega) + \sum_{k\in\mathcal{K}} \beta_{jk} n_{jk} F^s_{jkt}(\omega) \right) \right.$$

$$\left. - \sum_{j\in\mathcal{J}}\sum_{t\in\mathcal{T}}\left(h\bar{I}^N_{jt}(\omega) + aZ_{jt}(\omega) \right) - \sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{J}}\sum_{t\in\mathcal{T}} \delta W_{ijt}(\omega) + \sum_{j\in\mathcal{J}}\sum_{k\in\mathcal{K}}\sum_{t\in\mathcal{T}} \gamma F^n_{jkt}(\omega) \right) \tag{21}$$

$$-\left(\sum_{j\in\mathcal{J}} f_j X_j + \sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{J}} g_{ij} V_{ij} + \sum_{j\in\mathcal{J}}\sum_{k\in\mathcal{K}} l_{jk} Y_{jk} \right)$$

s.t: Contraints (2) – (5), $i \in \mathcal{I}, j \in \Im, k \in \mathcal{K}$

Contraints (7) – (20). $e \in \{s, n\}, i \in \mathcal{I}, j \in \Im, k \in \mathcal{K}, t \in \mathcal{T}, (\omega, \xi) \in \Omega^N$

3. Illustrative example

A planning horizon, assumed to include $T = 24$ weeks, is used. The ordering cost is set to 10 dollar per order. We assumed sales price (product value) for all demand zones to be equal to $\rho_k = 40$ and material purchase cost is $\delta = 12$. As assumed before, DZs cannot directly be served from sources, so the correspondent transporting is not allowed. The instances are designed with two sources, four potential DCs and 15 demand zones with weekly compound demand. Based on the realistic industrial problems examined, the number of demand zones in the problems is much larger than the number of potential DCs. The fixed opening cost of a DC is randomly generated in [1.9, 2.1]×10^6. Two types of the inventory holding cost are assumed, 0.5 per unit per week and 2.5 per unit per week. A Normal-Bernoulli compound demand for DZs and a triangular distributed lead time for DCs are assumed. For each instance size, we considered a network including two market segments, large-size DZs (LS) and medium/small-size DZs (MS) with compound Bernoulli-Normal demand process. We assumed that the LS includes 40% of the network where issues order with the rate of 0.65 and that the MS includes the rest of market where issues order with the rate of 0.50 (here, rate of issuing order refers to Bernoulli parameter). In order to show the unstable customer demand scenarios generated following Normal Distribution, demand mean-variance ratio is set to $\mu/\sigma^2 = 1$ for each demand type over periods. For the stochastic lead time, a triangular distribution function Tr(1;5;10) is used. To show the impact of stochasticity of the market demand on the SC design, we applied two approaches, one is considering the expected mean and standard deviation of demand, and the other is a stochastic demand generation via the Monte-Carlo procedure. The latter is called as stationary demand process, where the demand of each zone follows a compound demand distribution whose parameters are constant during the planning horizon. In this case, for each DZ per period, 20 demand values are generated. In the following the results of sample problems with low and high inventory costs are illustrated. It is shown in Table 1 although the number of opened DCs with the expected demand value and the multiple-scenario based model at a predefined inventory costs is the same, different allocation for each DC is observed. It also represented that at the higher inventory holding cost, the network allocation decisions with expected demand value is close to the multiple-scenario based model. This similarity is a bit less at the lower inventory holding cost. It is also found out that at the high inventory holding cost, the percentage of DZs multiple-sourced is much higher than the case with low inventory holding cost.

Table 1. SC Network analysis

Inventory cost	Demand type	# of Opened DCs	Allocation Similarity	% of DZs Single-sourced	% of DZs Double-sourced
High	Expected value	4	82%	47%	53%
	Multiple-Scenario	4		60%	40%

Low	Expected value	4	94%	88%	12%
	Multiple-Scenario	4		80%	20%

4. Conclusion

In this paper, a new two-stage mathematical model for a location-inventory problem has been studied in which the demand and lead-time are considered as uncertain parameters. The results are verified by comparing the problem with the expected demand value and multiple-scenario based model. It has been shown that considering these two modelling approaching has an impact on the SC network design. In addition, the impact of different inventory costs are also investigated on the SC network location and allocation decisions. It is worth noting that CPLEX is able to solve the small scenario instances in a reasonable computation time. We also experimented several instances and found out that solution computational time is highly dependent on the number of scenarios. As the complexity of the problem grows when the size of the instances increase, proposing a solution approach can be an extension of this problem. Our current experiments aim to provide more general insights on the impact of inventory decisions on the SC network design. In addition, our current work is on the development of a heuristic solution approach that can handle this solvability issue.

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