Production planning optimization in the wood remanufacturing mills using multi-stage stochastic programming

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Abstract. Wood remanufacturers grapple with several challenging characteristics. One of the most important difficulties in this industrial sector today concerns how to maintain promised service levels in highly dynamic market. To include the stochastic nature of demand in the production planning, multi-stage stochastic programming models are proposed and tested in this study. Our preliminary results show that the solution of a multi-stage model has the potential to significantly improve the performance of traditional planning models at a relatively low cost.

Keywords: multi-stage stochastic programming, uncertain demand, co-production, wood remanufacturing industry

1 Introduction

This study is motivated by a real-scale case in the wood remanufacturing industry, which transforms pieces of lumber into bed frame components. Production planning is quite a complex task in this context, since from a given piece of lumber many types of products can be produced while following a divergent co-production logic that cannot be avoided. Moreover, a given component can be produced using different alternative processes (consuming different types of lumber and producing different sets of co-products). These alternative processes are differentiated according to the production yield, required inputs, set of co-products, production times, and production costs. Furthermore, the market is highly dynamic, having a wide range of products, with short order-cycle times, and production needs to be planned according to a make-to-order philosophy in an environment with unreliable demand. These characteristics cause complexity in production planning environments of such mills and lead wood remanufacturers to dynamic plans. In this context, it is difficult to keep the promised service level at low costs.

The literature reports that stochastic programming approaches can efficiently deal with uncertainty in production planning, especially when dealing with highly variable demand. Most existing stochastic programming models in production planning assume a two-stage model [1, 2]. In two-stage stochastic programming, the decision process takes place in two stages. In the first stage, actions tackle uncertainty and in the second stage the corrective actions are chosen after the realization of the random variables. Multi-stage stochastic programming approaches have been applied to deal with uncertain demand in several areas, such as capacity planning [3, 4] and lot sizing [5, 6], just to mention a few. Although stochastic programming has also been used in the area of planning optimization in the forest products industry [7, 8], it is still an open research field in this industrial sector.

According to recent research in the forest value chain by [9], representing uncertainty through scenarios for operational problems is essential in forestry. The underlying uncertainty needs to be well represented in a manageable and feasible model. The literature regarding production planning in the wood remanufacturing industry is limited. A few researches propose lean manufacturing models [10], and production planning models [11, 12, 13]. Even though the inherent characteristics of softwood remanufacturing differentiate it from other plants in the forest products industry and make this sector even more complex to manage, the application of stochastic programming has been missed.

Thus, the objectives of this paper are: 1) to propose a multi-stage stochastic programming model for demand-driven production planning under uncertain demand for the remanufacturing sector; 2) to perform some preliminary tests in a real-scale industrial context and evaluate if this approach is really superior to the current approaches and estimate the possible performance gains; 3) to evaluate its computational costs in order to assess whether the new approach can be employed in a real industrial environment. In order to do so, we suppose that demand uncertainty evolves during the planning horizon as discrete time stochastic

process. As a result, the uncertainty is represented through a scenario tree and an objective function is chosen to represent the risk associated with the sequence of decisions to be made. The stochastic model aims to provide a production plan that is technically possible to be implemented while taking into consideration the possible demand scenarios and delivering a full recourse action in the future.
The remainder of the paper is as follows. In Section 2, we propose a multi-stage stochastic program for wood remanufacturing production planning with uncertain demand. Experimentation and computational results are presented in Section 3. Section 4 concludes the paper and discusses possible future works.

2 A multi-stage stochastic program with uncertain demand

Most practical decision problems involve a sequence of decisions that respond to business conditions that evolve over time. Multi-stage stochastic programming approach is proposed to address optimization models in multiple periods while the uncertainty is revealed, and decisions are taken at every stage. In the following, we first describe the scenario tree, and then provide mathematical formulations for multi-stage stochastic programming.

2.1 Modeling the uncertain demand

We suppose that demand uncertainty evolves during the planning horizon as discrete time stochastic process. Scenario trees are common structures to show how uncertainty unfolds over time as the possible sequences of data are depicted. A common scenario tree structure includes regular scenario tree wherein all nodes have the same number of child nodes. We consider three possible market conditions for products demand; namely, High, Average, and Low to generate a regular trinomial scenario tree where each node has three child nodes (branches) with High, Average, and Low random demands. Demand data is fitted by normal distribution probability function using statistical tests. Thus, a three-point discrete distribution can be approximated by the Gaussian quadrature method [14]. To present the random demand as a scenario tree, the planning horizon is divided into stages. Each stage shows the step of time when new information on the random demand is available to decision maker. According to the case condition, we also assume that the demands for all products are perfectly correlated and have the same market condition at each stage of the scenario tree. Moreover, it is supposed that the decision maker is perfectly aware of the demand scenario at the start of each stage. As the availability of information on the uncertain parameter at the start of each stage in the scenario tree is perfect, a full recourse action is considered for this uncertain parameter in the multi-stage stochastic model.
As we cluster the 54-period planning horizon into 4 stages, the first stage consists of time period zero (current time), the second stage includes periods 1-18, the third stage consists of periods 19-36, and finally the fourth stage includes periods 36-54. We suppose that if at stage *i* the market is booming, the demand scenario for all products can be expected to be High. If the market is steady, the demand scenario for all products can be expected to be Average. In sluggish and weak market conditions, the demand scenario for all products can be expected to be Low. Such clustering results in a scenario tree with 27 demand scenarios and 40 nodes.

2.2 Multi-stage stochastic programming model

In this section we propose a multi-stage linear programming model for production planning in wood remanufacturing mills. The original deterministic model was proposed in [11]; however, we extend it through adding a different optimization approach. The new model (1A)-(9A) is in Appendix A.

2.2.1 Notations

The following notations are used for:

Sets

$P^{consumed}$ Products p that can be consumed

$P^{produced}$	Products p that can be produced
T	Set of periods in the planning horizon, $t \in T$ is an index
R	Set of recipes r (A recipe is called an alternative process)
ST	Scenario tree
n, n'	Nodes of the scenario tree $n, n' \in ST$
$predec(n)$	Predecessor of node n in the scenario tree
t_n	Set of time periods corresponding to node n in the scenario tree (number of periods in one stage)

Parameters

c_{rt}	Production costs associated with using recipe r in period t
i_{pt}	Inventory holding cost per unit of products $p \in P^{produced}$ in period t
bo_{pt}	Backorder cost per unit of product $p \in P^{produced}$ in period t
δ_r	Capacity required for each recipe r per unit time
c_t	Available capacity for period t (number of time units)
ic_{p0}	Inventory of material $p \in P^{consumed}$ at the beginning of planning horizon
s_{pt}	Supply of raw material $p \in P^{consumed}$ provided at the beginning of period t
ip_{p0}	Inventory of product $p \in P^{produced}$ at the beginning of planning horizon
φ_{pr}	The units of raw material $p \in P^{consumed}$ consumed by recipe r
ρ_{pr}	The quantity of product $p \in P^{produced}$ produced by recipe r
r_{pr}	Selling price per unit of product $p \in P^{produced}$ according to recipe r
$d_{pt}(n)$	Demand of product $p \in P^{produced}$ to be delivered by the end of period t at node n of the scenario tree
$prob(n)$	Probability of node n of the scenario tree

Decision variables

$X_{rt}(n)$	Control variable - Number of times each recipe r should be run in period t at node n of the scenario tree
$IC_{pt}(n)$	Inventory size of raw material $p \in P^{consumed}$ by the end of period t at node n of the scenario tree
$IP_{pt}(n)$	State variable - Inventory size of product $p \in P^{produced}$ by the end of period t at node n of the scenario tree
$BO_{pt}(n)$	State variable - Backorder size of product $p \in P^{produced}$ by the end of period t at node n of the scenario tree
$F_{pt}(n)$	Quantity of sold product $p \in P^{produced}$ by the end of period t at node n of the scenario tree

2.2.2 The multi-stage model

A multi-stage stochastic model is formulated based on the scenario tree for the uncertain demand in this section. The control variable of model (1)-(7) is production plan X_{rt}. The state variables of the plan are the inventory quantity variable (IP_{pt}) and the backorder quantity variable (BO_{pt}). As we suppose that the decision maker (planner) is aware of which demand scenario is forced for the stage, the multi-stage stochastic model is a full recourse with regard to demand scenario. As a result, the decision variables X_{rt}, the inventory quantity variables IP_{pt} and the backorder quantity variables BO_{pt} for each node of scenario tree are defined to present the model.

$$Maximize \sum_{n\in ST} prob(n)\left(\sum_{t}^{T}\sum_{r\in R}\sum_{p\in P^{produced}} r_{pr}F_{pt}\right) - \sum_{n\in ST} prob(n)\left(\sum_{t}^{T}\sum_{r\in R} c_{rt}X_{rt} + \sum_{t\in t_n}^{T}\sum_{p\in P^{produced}} (inv_{pt}IP_{pt}(n) + bo_{pt}BO_{pt}(n))\right) \quad \textbf{(1)}$$

$$\sum_{\forall r\in R} \delta_r X_{rt}(n) = c_t \qquad \forall \ t\in t_n \ , n\in ST \quad \textbf{(2)}$$

$$IC_{pt}(n) = IC_{pt-1}(n') + s_{pt} - \sum_{\forall r\in R} \phi_{pr} X_{rt}(n) \quad \textbf{(3)}$$

$$\forall p\in p^{consumed}, \quad t\in t_n, n, n'\in ST, \quad n' = \begin{cases} n & if \ t-1\in t_n \\ predec(n) & if \ t-1\notin t_n \end{cases}$$

$$IP_{pt}(n) - BO_{pt}(n) = IP_{pt-1}(n') - BO_{pt-1}(n') + \sum_{\forall r\in R} \rho_{pr} X_{rt}(n) - d_{pt}(n) \quad \textbf{(4)}$$

$$\forall p\in p^{produced}, \quad t\in t_n, \ n, n'\in ST, \quad n' = \begin{cases} n & if \ t-1\in t_n \\ predec(n) & if \ t-1\notin t_n \end{cases}$$

$$\sum_{t}^{T}\sum_{p\in P^{produced}} F_{pt}(n) \le d_{pt}(n) \qquad \forall p\in p^{produced}, \ t\in t_n, n\in ST \quad \textbf{(5)}$$

$$X_{rt}(n)\ge 0, \ IC_{pt}(n)\ge 0 \qquad \forall p\in p^{consumed}, t\in t_n, \ n\in ST, r\in R \quad \textbf{(6)}$$

$$IP_{pt}(n)\ge 0, \ F_{pt}(n)\ge 0, \ BO_{pt}(n)\ge 0 \qquad \forall p\in p^{produced}, \ t\in t_n, n\in ST \quad \textbf{(7)}$$

The objective function in (1) consists in maximizing the expected profit, which is the difference between total revenue and total costs. Constraint (2) guarantees that the total production time does not exceed the available time and production capacity. Constraints (3)-(4) ensure flow equilibrium of raw materials and final products. Constraint (5) guarantees that sales do not exceed customer demand. Constraints (6)-(7) enforce the non-negativity on the decision variables. The decision variables in the model (1)-(7) are indexed for each node and a set of periods in each stage denoted by t_n. To transfer the inventory and backorder quantities from one stage to another one, two variables are considered (BO_{pt} and IP_{pt}). These variables transfer the ending values of inventory and backorder of the previous stage to the first period of relevant nodes at the current stage. In other words, when the stage changes, the first period of the current node (n) takes the initial inventory and backorder quantities from the last period of the immediate predecessor node (n') in the previous stage. However, the other periods (except the first period) at the current node (n) in the given stage, the initial inventory and backorder quantities derive from previous periods at the same node (n).

Notice that machine reconfiguration (setup time) for switching from one recipe to another is constant in this model. This assumption makes the model feasible to compute an exact result. However, considering

binary decision variables related to the setup time might cause infeasibility to compute an exact result with a deterministic algorithm. Under this condition, a decomposition method i.e., Lagrangian relaxation or Bender decomposition would be adopted to solve the model.
Another point is that alternative approaches to deal with this problem include Markov decision processes and Approximate Dynamic Programming. Such approximation approaches may be useful for large problem instances.

3 Preliminary computational results

A wood remanufacturing mill in Eastern Canada is selected as a case study. The planning horizon consists of 54 periods.
For random demand data, we use normal distribution according to statistical tests results. We also compare the solutions of three scenario trees for 4-stage, 3-stage and 2-stage models to show the differences between dynamic recourse models and static resource one. To describe the demand evolution over the planning horizon, four demand patterns (DP) are considered with the same mean but different standard deviations (DP1:5% mean, DP2: 10% mean, DP3: 20% mean, and DP4: 30% mean). Considering one planning approach, demand patterns (4 patterns), scenario trees (3 trees), and repetitive numerical experiments (30 runs), we generate a total number of 360 problems and solve them. To solve the proposed model, CPLEX 12.6.1 and OPL 6.2 are used. All numerical experiments are conducted on an Intel® Core™ i7-4700HQ processor, 2.40GHz, 12 GB of RAM, running Microsoft Windows 8.1.
In Table 1, we compare the solution of a 4-stage stochastic programming model to those of a 3-stage model, 2-stage model, and deterministic equivalent model for the four demand patterns with respect to the expected profit, the expected inventory/backorder costs and CPU times. As can be observed, the profits of the 2-stage models are less than those of the multi-stage stochastic models and by increasing demand variability at each stage (from DP1 to DP4), the differences between the profit of plans in multi-stage stochastic programming models become bigger from the plan of two-stage stochastic models and deterministic equivalent models. Moreover, by increasing the number of stages the inventory/backorder costs also decrease. Along with that, the table clearly indicate that in each stage, the increase in demand variability (from DP1 to DP4) results in increase the inventory/backorder costs. The CPU column shows that the high quality of multi-stage stochastic model requires higher computational time compared to those of the two-stage ones; however, it still has reasonable time. As a result, multi-stage models have better performance while demand variability increases.

Table 1: Results comparison of different production planning models

Demand pattern	Production planning model	Expected profit	Expected INV/BO costs	CPU time (seconds)	Possible gains (expected profit)	Possible gains (Expected costs)
DP1	4-Stage SP	111,581	48,726	824	_	_
	3-Stage SP	82,510	49,044	252	35%	1%
	2-Stage SP	60,840	51,033	93	83%	5%
	Deterministic	35,340	52,574	0	216%	7%
DP2	4-Stage SP	99,613	49,058	825	_	_
	3-Stage SP	76,448	51,136	250	30%	4%
	2-Stage SP	52,826	52,175	91	89%	6%
	Deterministic	32,660	52,988	0	205%	7%
DP3	4-Stage SP	91,075	51,800	826	_	_
	3-Stage SP	71,981	52,104	254	27%	1%
	2-Stage SP	48,494	52,935	92	88%	2%
	Deterministic	29,588	53,494	0	208%	3%
DP4	4-Stage SP	82,257	67,404	823	_	_
	3-Stage SP	64,843	73,060	251	27%	8%
	2-Stage SP	42,879	80,132	93	92%	16%
	Deterministic	27,985	84,938	0	194%	21%

Figure 1 graphically presents some results of Table 1. Generally, since these preliminary results indicate that the solutions of multi-stage stochastic models are better than the deterministic models, this should come as no surprise. The expected profits of the 2-stage models are less than those of the multi-stage stochastic models; however, their results are far superior to those of the deterministic models. In addition, it can be noted that significant gains can be obtained in terms of expected profits (up to 216%) and expected costs (up to 21%). Most important, this is obtained at a computational cost of less than 14 minutes of CPU time with a standard machine, which makes this model fairly useful in a real-industrial environment where decision makers need fast decision support systems. We did not regard more stages in the scenario trees because the difference between the 3-stage and 4-stage models is not very significant.

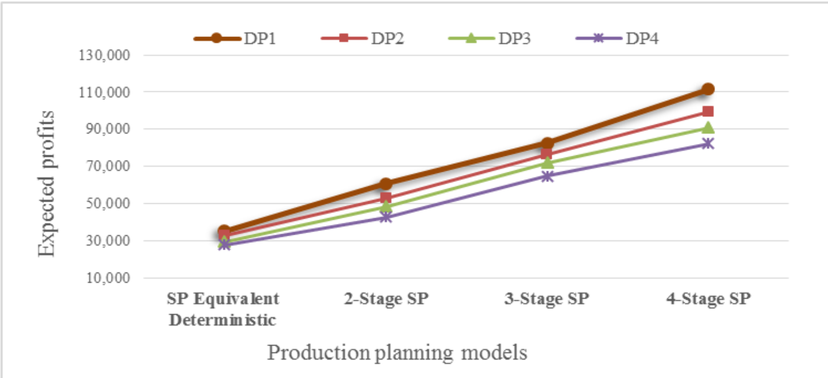

Figure 1: Expected profits for different planning models

3.1 Quality of stochastic solutions

To compare the value of the two-stage and multi-stage models in different demand patterns, we evaluate the "Value of Multi-stage Stochastic Programming" (VMS) proposed by [15]. The VMS is defined as $v^{MS} - v^{TS}$ where v^{TS} and v^{MS} are the optimum objective values of two-stage and multi-stage models, respectively. Figure 2 shows the VMS in four different demand patterns. The observation from Figure 2 is that the pattern of demand distribution has an influence on the magnitude of the VMS. In other words, as the variability of demand increases at each stage, applying a multi-stage stochastic model becomes more significant with respect to its maximization minimization objective.

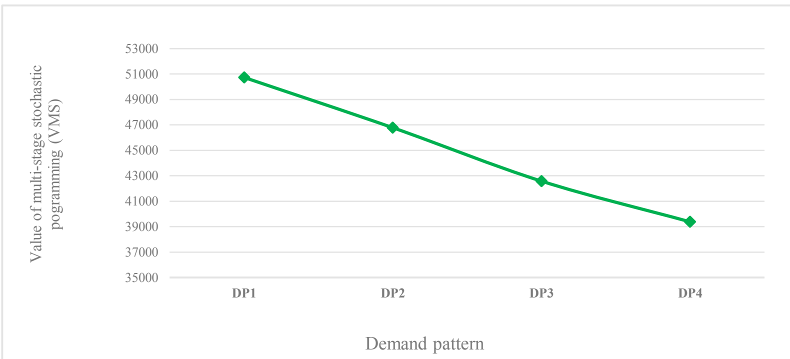

Figure 2: Comparison of different planning models in terms of VMS

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4 Conclusions

We propose a multi-stage stochastic model to address the problem in a real-scale wood remanufacturing mill. The numerical results confirm that the quality of the multi-stage stochastic model is better than those of the deterministic equivalent and two-stage models. Another observation is that, as the variability of demand increases at each stage, the difference between the expected profit of the multi-stage stochastic model and the deterministic and two-stage stochastic models increases and this proves the significance of using multi-stage under increase on demand variations. As further extensions of this study, adding setup time constraints in the model and solving the model with a decomposition method can be considered. For large problem instances, Markov decision processes and Approximate Dynamic Programming could be considered as alternative approaches. Robust optimization can be another extension for the production planning of the wood remanufacturing mills involving challenging characteristics.

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Appendix: Deterministic linear programming model

$$Maximize \ \sum_{t}^{T}\sum_{r\in R}\sum_{p\in P^{produced}} r_{pr}F_{pt} - (\sum_{t}^{T}\sum_{r\in R} c_{rt}X_{rt} + \sum_{t}^{T}\sum_{p\in P^{produced}} i_{pt}IP_{pt} + bo_{pt}BO_{pt}) \qquad (1A)$$

$$\sum_{\forall r\in R}\delta_r X_{rt} = c_t \qquad \forall t = 1,...,T \qquad (2A)$$

$$IC_{P1} = ic_{p0} + s_{p1} - \sum_{\forall r\in R}\phi_{pr}X_{r1} \qquad \forall p \in p^{consumed}, t = 1 \qquad (3A)$$

$$IC_{pt} = IC_{pt-1} + s_{pt} - \sum_{\forall r\in R}\phi_{pr}X_{rt} \qquad \forall p \in p^{consumed}, t = 2,...,T \qquad (4A)$$

$$IP_{p1} - BO_{P1} = ip_{p0} + \sum_{\forall r\in R}\rho_{pr}X_{r1} - d_{p1} \qquad \forall p \in P^{produced}, t = 1 \qquad (5A)$$

$$IP_{pt} - BO_{pt} = IP_{pt-1} - BO_{pt-1} + \sum_{\forall r\in R}\rho_{pr}X_{rt} - d_{pt} \qquad \forall p \in p^{produced}, t = 2,...,T \qquad (6A)$$

$$\sum_{t\in T}\sum_{p\in P^{produced}} F_{pt} \le d_{pt} \qquad \forall p \in P^{produced}, t = 1,...,T \qquad (7A)$$

$$X_{rt}, BO_{pt}, IP_{pt} \ge 0 \qquad \forall r \in R, \forall p \in P^{produced}, t = 1,...,T \qquad (8A)$$

$$IC_{pt} \ge 0 \qquad \forall p \in P^{consumed}, t = 1,...,T \qquad (9A)$$