Lumber supply chain tactical planning under demand and supply uncertainty

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Abstract. We propose a mathematical approach based on a multi-stage stochastic mixed-integer programming for lumber supply chain tactical planning under demand and supply uncertainty. Due to its large size for real instances, we decouple it into two sub-models inspiring from the common practice in industry. Each sub-model is then solved by a Scenario Cluster Decomposition (SCD) method. In order to speed-up solving cluster sub-problems, a Lagrangian-Relaxation based heuristic and a Variable Fixing algorithm are embedded into the SCD algorithm. Computational experiments on a real-size lumber supply chain clearly indicate the effectiveness of the proposed solution approaches in terms of CPU time and quality of solutions.

Keywords: Lumber supply chain; tactical planning; Multi-stage stochastic mixed-integer programming; Scenario Cluster Decomposition.

1 Introduction

Lumber supply chain (SC) incorporates forests, as suppliers, sawmills as the manufacturing entities, distribution channels, as well as contract and non-contract-based customers. Tactical planning in the lumber SC determines the optimal log harvesting/procurement, and lumber production/distribution/sale decisions over a planning horizon with the goal of maximizing SC profit. Unlike the convergent manufacturing industries, the lumber supply is characterized by a divergent structure and the highly heterogeneous nature of its raw material [1] as well as uncertain lumber demand in the market. The above-mentioned problem has been mainly addressed in the literature in a deterministic context (e.g., see [1-3]). Nevertheless, the plans proposed by deterministic approaches are not realistic and robust in the presence of future uncertain events [4]. To the best of our knowledge, only sawmill production planning under uncertainty was investigated in [5-9]. This is the main motivation of this research for proposing a stochastic tactical planning tool in this SC such that the plan is protected against random variations in supply and demand.

By modeling the uncertain demand as a dynamic stochastic parameter over the planning horizon, the lumber SC tactical planning problem is modeled as a multi-stage stochastic mixed integer program (MS-MIP). Nonetheless, while applying to real-size instances, such models are among the most intractable ones. A variety of algorithms for solving MS- MIP models have been proposed in the literature (e.g., see [8-13]). Scenario partitioning (clustering) in scenario trees is one of the most promising algorithms to speed up solving large-scale MS-MIP models (e.g., see [7-8], [11-13]).

In this article, in order to alleviate the complexity of the MS-MIP lumber SC tactical planning model, we first decouple it into two sub-models inspired by the current practice in industry. The latter sub-models are then solved subsequently by the aid of Hybrid Scenario Cluster Decomposition (HSCD) algorithm proposed in [13].

2 Problem description

The goal of lumber SC tactical planning is to maximize the global net profit by balancing the sale revenue and supply chain cost over a one-year planning horizon. The harvesting decisions involve the blocks where the harvesting should occur as well as the proportion of the harvested blocks in different periods of the planning horizon. The latter decisions must satisfy the block harvesting and transportation capacity, the maximum number of times each block can be harvested and the maximum number of blocks available to harvest over the planning horizon. The procurement planning decisions include the purchasing quantity of raw material from each block, and the inventory of raw materials in each block. Such decisions must satisfy the contract quantity commitment as well as inventory capacity of harvesting blocks. Production planning decisions incorporate the quantity of lumbers that should be sawn, dried, and finished as well as inventory and backlog quantities of lumbers. Production decisions must satisfy production and inventory capacity and yield of sawing, drying, and finishing processes. Distribution planning decisions include the shipping quantity of products, the inventory quantity of products in each distribution center, the number of truckload requirement along with the type of vehicle and route by considering transportation capacity. Finally, sale planning involves the amount of sale promised to customers as well as possible backlog quantity of products. The mathematical formulation of lumber SC tactical planning is provided in [2].

In this study, we assume that the uncertain lumber demand has a non-stationary behavior over the planning horizon; hence it is modeled as a scenario tree. On the other hand, the uncertainty in the availability of logs in each block of the forest is assumed time-independent during the planning horizon and modeled as a scenario set. The uncertain log supply is then integrated into the demand scenario tree by considering a number of scenarios for the uncertain supply in each node of the demand scenario tree. Consequently, the integrated tactical planning problem can be formulated as an MS-MIP model. In this model, the harvesting, procurement, production, distribution, and sale decisions described above are defined for each node of the demand scenario tree while the inventory and backlog decisions are defined for each demand node and each log supply scenario. The objective function maximizes the expected profit for all demand nodes and supply scenarios. More details on stochastic lumber supply chain tactical planning model are provided in [13].

3 Solution methodology

It should be noted that the aforementioned MS-MIP model is non-tractable for real-size instances. Hence, inspired by the current practice in industry, we propose to decouple it into the following two sub-models: 1) the production/distribution/sale (PDS) sub-model (a multi-stage stochastic linear program (MS-LP) that deals exclusively with uncertain demand; and 2) the harvesting/procurement (HP) MS-MIP sub-model that is affected by uncertain log supply in addition to log demand. The PDS and HP sub-models are accordingly solved in a sequential manner. It has been demonstrated in [2] that such a decoupling approach leads to a small gap in terms of SC profit comparing to the integrated planning method in a deterministic context. Further, in order to avoid infeasible plans in the upstream echelon (i.e., HP sub-model), it is necessary to add extra coupling constraints from the HP sub-model to the PDS sub-model. Such constraints enforce the PDS sub-model to control the production amount based on the supply and transportation capacity of raw material in the forest. Also, they ensure that the production amount satisfies the minimum purchase quantity of raw material from each harvesting block of the forest. The PDS and HP sub-models are both large-scale MS-LP and MS-MIP ones that are hard to solve for large instances. Thus, we propose to apply the scenario cluster decomposition (SCD) and hybrid scenario cluster decomposition (HSCD) algorithms [13] to respectively solve each sub-model. Figure 1 summarizes the key steps of our proposed methodology to solve multi-stage stochastic lumber supply chain tactical planning model.

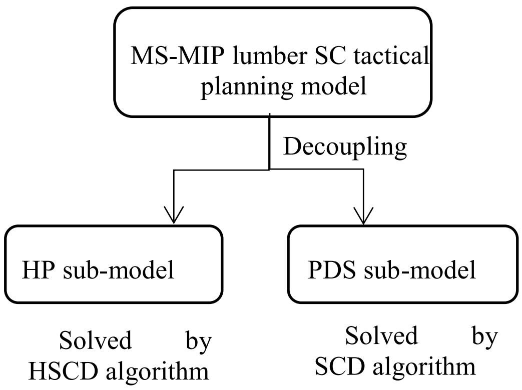

Fig 1- Solution methodology for multi-stage stochastic lumber SC tactical planning model

In what follows, we elaborate on the main steps of SCD and HSCD algorithm.

Scenario cluster decomposition (SCD) algorithm

The idea is to decompose the initial scenario tree into smaller sub-trees that share a certain number of ancestor nodes. The multi-stage stochastic model is then decomposed into scenario cluster sub-models which are coordinated by Lagrangian penalty terms in their objective function in order to compensate the lack of non-anticipativity condition (NAC). The latter condition requires that the decision variables corresponding to each node of the scenario tree in each stage are identical for any pair of indistinguishable scenarios at that stage. The sub-gradient algorithm (SA) is then implemented to coordinate scenario cluster sub-models into an implementable solution. It should be noted that NACs enforce implementability condition for the main decision variables in the PDS and HP sub-models, i.e., harvesting, procurement, production, distribution, and sale decisions. It should be noted that scenario cluster sub-models can either solved to optimality by a commercial solver or can be solved by an efficient heuristic in order to obtain a near optimal solution in each iteration of sub-gradient algorithm. In this study, the PDS sub-model is a multi-stage stochastic LP model. Hence, scenario cluster sub-models corresponding to this problem can be solved by a solver in a relatively short CPU time. On the contrary, the HP sub-model is a MS-MIP model and its scenario cluster sub-models are solved by the aid of a Lagrangian Relaxation heuristic. Hence, the HSCD algorithm is applied to solve it.

Hybrid scenario cluster decomposition (HSCD) algorithm

This heuristic is similar to the SCD algorithm described above except that an ad-hoc heuristic is used to solve scenario cluster sub-models. Also, a variable fixing heuristic is also embedded within the SCD algorithm so as to accelerate the convergence of sub-gradient algorithm. The main steps of this algorithm are as summarized in algorithm 1. The comprehensive description of HSCD algorithm along with mathematical notations is provided in [13].

Algorithm 1. HSCD algorithm

Step 1. Scenario Cluster Decomposition (SCD) algorithm
- *Step 1.1. Partition the scenario tree into a number of sub-trees*
- *Step 1.2. Formulate scenario cluster sub-models in a compact form after adding NAC violation terms in their objective function*

Step 2. Solving scenario cluster sub-models (Sub-gradient algorithm)
- *Step 2.1. Ad-hoc heuristic (to solve each scenario-cluster sub-model)*
- *Step 2.2. Variable-Fixing heuristic (VFH) (to speed-up the sub-gradient algorithm)*

Solving scenario cluster sub-models

Due to the computational complexity of the HP sub-model, we solve its scenario cluster sub-models by the aid of a Lagrangian Relaxation Heuristic (LRH) proposed in [2]. In this algorithm, harvesting constraints that contain binary variables are relaxed and corresponding penalty terms are added to the objective function in order to obtain the Lagrangian Relaxation sub-model. With the goal of accelerating the SA applied to solve the Lagrangian model and to guarantee the feasibility of converged solution, the LRH heuristic updates the upper-bound on the objective function value of the HP sub-model based on the most recent lower-bound obtained in each iteration of the SA. This will help to better adjust the step-size of SA.

Variable-fixing heuristic(VFH) to speed-up the SCD algorithm

In order to accelerate the convergence of the SA within the SCD algorithm (step 1), VFH [13] is implemented. The idea behind this algorithm is to fix the value of some binary variables to 1 in each iteration of SCD algorithm according to a consensus rule among the solution of all scenario cluster sub-models.

4 Numerical results

In this section, we provide the results of applying the SCD and HSCD algorithms to lumber supply chain tactical planning under demand and supply uncertainty in a real-size case study in Canada. All parameters of the lumber supply chain are inspired from [2, 3]. We also assumed that the lumber demand for the next 3 periods (months) has a stationary behavior. Thus, the 12 periods planning horizon is clustered into four stages which will result a five-stage multi-stage stochastic model. At each stage, we consider a normal distribution for the demand which is approximated by a 3-point discrete distribution. In each node of this scenario tree, 3 scenarios have been considered for the log availability in harvesting blocks. In order to better validate the HSCD algorithm, 3 test problems were generated by considering 3 demand scenario trees with the same average demand and different variances (i.e., 5%, 20% and 30% of the average). All algorithms in this paper were coded in C++ using CPLEX v12.5 concert technology on a Dual-Core CPU 3.40GHz computer with 16.00 GB RAM.

Table (1) provides the results of SCD algorithm with 9 sub-trees for the PDS sub-model. All profit values are divided by 1000, and CPU times are provided in seconds. "iter" represents the number of iterations that the HSCD algorithm needs to reach a converged solution. The "Gap%" is the relative gap between the best feasible solution found by the HSCD algorithm and the optimal solution found by CPLEX. As can be observed in this table, the HSCD approach provides high quality solutions with small (negligible) optimality gaps in a reasonable time. Also, as the variability of demand increases (test problem 3), the quality of solution slightly deteriorates. This is due to higher differences between the solution of scenario cluster sub-models.

Table 1. SCD algorithm with 9 sub-trees for PDS sub-model

	Profit (HSCD)	Iter	Profit (CPLEX)	HSCD time	CPLEX time	Gap%
Test problem 1	623,673	4	623,661	1,713	838	0.0019
Test problem 2	623,488	3	623,462	1,883	961	0.0041
Test problem 3	623,374	4	623,342	1,845	898	0.005

Table (2) represents the results of applying HSCD algorithm to the HP sub-model when we decompose it into 9 clusters. In this table, we also provide the results with and without implementing the LRH and VFH heuristic algorithms. While CPLEX is unable to solve the HP sub-model within 24h CPU time, the HSCD algorithm can solve this complex model in less than 2h. Furthermore, as can be observed in Table (2), applying the LRH and VFH heuristic algorithms in the HSCD algorithm considerably improves the CPU time (average of 85% over all test instances). Furthermore, considering VFH heuristic in addition to LRH improves the CPU time by 19% on average. These results clearly indicate the superiority of the HSCD algorithm over a commercial solver such as CPLEX in solving large-scale multi-stage stochastic mixed integer programs similar to the one for integrated tactical planning in the lumber supply chain.

Table 2. HSCD algorithm with 9 sub-trees for HP sub-model

Test problem	Algorithm	Cost	CPU time	Iteration
1	SCD with LRH	27,635	10,844	3
1	HSCD	27,636	8,391	3
1	SCD	27,628	62,648	5
2	SCD with LRH	27,935	7,604	4
2	HSCD	27,946	6,562	2
2	SCD	27,934	45,500	6
3	HSCD with LRH	28,451	9,328	3
3	HSCD	28,451	7,535	3
3	SCD	28,447	41,276	5

5 Concluding remarks

In this study, we proposed a mathematical approach based on multi-stage stochastic programming for lumber supply chain tactical planning. In order to tackle the complexity of the aforementioned model for real-size instances, we proposed a decoupling strategy such that the original MS-MIP model into smaller MS-LP and MS-MIP models such inspired by the practice in industry. Additional linking constraints were also added from the upstream sub-model (i.e., harvesting/procurement sub-model) to the downstream one (i.e., production/distribution/sale sub-model) so as to avoid infeasibility. Further, with the goal of speeding the solution process, rather than using a commercial solver to solve each decoupled model, we applied two state-of-the-art heuristics in the literature [13], i.e., SCD and HSCD algorithms, to efficiently solve the aforementioned models. Our experimental results on a set of realistic-size cases revealed that the HSCD algorithm can find high quality solutions in less than 2h CPU time while CPLEX fails to find the optimal solution with 24h.

While lumber industry is faced with supply and demand perturbations, adopting a stochastic optimization approach in the existing decision models seems to be inevitable for robust decision making In contrast, such models are featured as intractable ones for real-size problem instances. While the ability to come up with robust plans in a relatively short amount of time is one of the main competitive advantages of an industry, the SCD and HSCD algorithmic procedure applied in this paper is an attempt to reduce the challenge of solving such problems.

6 References

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