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A stochastic programming approach for closed-loop supply chain network design under uncertain quality status

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Abstract. In this article, we presents a two-stage stochastic programming model for the problem of designing a closed-loop supply chain network which is applicable in the context of modular structured products. The model accounts for uncertainty in the quality status of the return stream, modeled as binary scenarios for each component in the reverse bill of material corresponding to such products. The stochastic model is solved by the aid of an accelerated L-shaped algorithm by considering a reduced set of scenarios.

Key words: Closed-loop supply chain; Durable products; Uncertain quality state

1 Introduction

In response to sustainability of supply chains, design and management of closed-loop supply chains (CLSC) have attracted a growing interest over the recent decade. It has been recognized that CLSCs comprise forward channel along with the so-called "reverse supply chain (RSC)". Designing a CLSC network for durable products, that are characterized by their modular structured design and their long life cycle (e.g., large household appliances), is a complex problem. Such category of products can be disassembled into several components (i.e., modules and parts) as well as raw materials concerning the reverse bill of materials (BOM). We note that in the context of CLSC/RSC network design, most studies are limited to involving a few recovery practices, e.g., remanufacturing and material recycling, in designing their networks.

A clear distinction that is made between CLSCs and the forward supply chains lies in uncertain condition (quality) of cores. On the other hand, the overview of the existing literature implies that in most of prevailing studies uncertainty consideration is limited to demand and quantity of returns. However, the impact of uncertain quality state of returns on CLSC design, regardless of its substantial impact, has not been adequately investigated in the literature

[2, 3, 4, 7, 9]. The analysis of the current literature suggests that the random quality status has been roughly estimated, leading to a small number of scenarios representing this uncertain aspect.

Observing the above-mentioned drawbacks, in this study, we address various recovery options, which an OEM can adopt in tackling the return stream. These recovery processes are plausible in taking different sub-assemblies of a typical durable product. We also propose a precise approach to model the uncertain quality status, where the availability of each component in the reverse BOM is modeled as a discrete scenario following a Bernoulli distribution. In this regard, we formulate this large-scale optimization problem as a two-stage stochastic mixed-integer program with recourse [1]. As far as the authors are aware, design of a CLSC concerning the explicit modeling of the uncertain quality of each component in the reverse BOM has never been investigated in the context of durable products.

Since the aforementioned approach for modeling the uncertain quality exponentially increases the number of scenarios, we implement fast forward selection algorithm [5], adapted to our problem setting in [6], as a scenario reduction technique to preserve the most pertinent scenarios. Further, we apply an L-shaped algorithm [10] as described in [6].

2 Problem description and formulation

2.1 CLSC network design for durable products

In our problem of interest, an organization operates a well-established supply chain in which the forward network comprises components and raw materials suppliers, manufacturing facilities, distribution centers, and end-user locations. The aim is to extend the existing forward network to accommodate the recovery facilities and consequently to coordinate the physical forward and reverse flows in the extended supply chain network. The reverse network includes collection, disassembly, remanufacturing, bulk recycling, material recycling, and disposal centers. The returned durable products received at end-user zones are shipped to disassembly centers through collection centers. In disassembly centers, the inspected return stream is disassembled into different components based on the reverse BOM. The recoverable modules and recyclable materials are then sent to remanufacturing and recycling centers for further processes. Besides, the bulk of mixed residues would typically be processed in bulk recycling centers and/or outsourced to a third-party provider to separate the precious raw materials from mixed scrap, e.g., electronic scrap. The bulk recycling step is followed by material recycling and safe disposal. The remanufactured modules, spare parts, and recycled raw materials are either shipped to manufacturing facilities to deploy in manufacturing of brand-new products and/or sold on secondary markets. Given the above description, the conceptual structure of the CLSC under consideration is schematically illustrated in Figure 1. The solid arcs indicate the forward flows while the dashed ones denote the reverse flows in the CLSC.

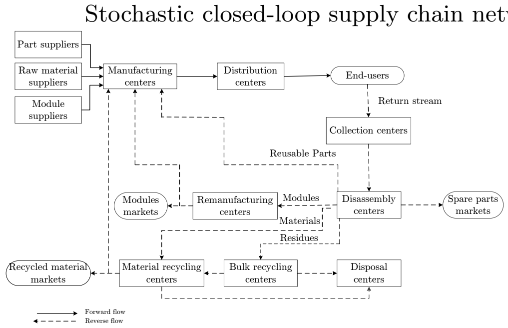

Fig. 1. General structure of the CLSC network

2.2 Modeling random quality states of the return stream

Due to the usage and the deterioration rates during the long life cycle of durable products, in many cases, it is quite impossible to foresee the exact number of recoverable components in a returned durable product. Moreover, the quality status can only be revealed after grading the returned items in disassembly centers. In this study, the random quality status is defined as the availability of each component in the reverse BOM and modeled as discrete scenarios with Bernoulli probability distribution. Let P and L denote, respectively, the set of parts and modules in the brand-new durable product. Further, let γ_p, δ_l, and β denote, respectively, the number of reusable part p, the number of remanufacturable module l, and the mass of residues in the returned durable product. Now that we represent the random quality vector by $\boldsymbol{\xi}$ where $\boldsymbol{\xi} = \{\gamma_p | \forall\, p \in P; \delta_l | \forall\, l \in L; \beta\}$. We also represent each particular realization (scenario) of the random quality status by $\gamma_p(\boldsymbol{\xi}_s)$, $\delta_l(\boldsymbol{\xi}_s)$, and $\beta(\boldsymbol{\xi}_s)$. Each particular scenario s is associated with a non-negative probability π_s such that $\sum_{s \in S} \pi_s = 1$. Once the grading process is executed in disassembly centers, it is realized whether or not a particular part/module is suitable for the effective recovery process. We assume that the grading process yields a good condition component with success probability $\hat{p}$ otherwise a poor state one with a failure probability $\hat{q} = 1 - \hat{p}$. It should be emphasized that the success probability of each component (module/part) is the same in all returned items. Moreover, we assume that the probability distribution corresponding to the condition of different returned items are independent and identical.

For instance, we point out to a used twin tub of a washing machine such that each washing tube unit weighs 3.5 kg. Every unit can independently be either functional with probability $\hat{p}$ or defective with probability $\hat{q}$. Hence, the outcome of the grading process for the washing tube follows a Bernoulli distribution. Likewise, the quality status of other components in the reverse BOM is an independent random variable following a Bernoulli distribution. For every realization of the random quality vector, i.e., $\boldsymbol{\xi_s}$, we define an indicator function for each unit j of part p and another indicator function for each unit k of module l as follows.

$$I(p^j) = \begin{cases} 1 & \text{if unit } j \text{ is in a functional state;} \quad j = 1, ..., n^p \\ 0 & \text{otherwise} \end{cases}$$

$$I(l^k) = \begin{cases} 1 & \text{if unit } k \text{ is in a functional state;} \quad k = 1, ..., n^l \\ 0 & \text{otherwise} \end{cases}$$

It allows us to consider, respectively, the number of reusable part p as $\gamma_{ps} = \sum_{j=1}^{n^p} I(p^j)$ and the number of remanufacturable module l as $\delta_{ls} = \sum_{k=1}^{n^l} I(l^k)$. For example, in the aforementioned washing machine case, a possible outcome of the grading process might be one fully functional and one defective washing tube. Thus, for this specific part, γ is equal to 1. On the other hand, the indicator function of the defective unit takes 0 with probability $\hat{q}$. The defective unit will considered as residues and the viable recovery option for this unit will be bulk recycling process. In this regard, β will be increased with the corresponding weight of the unit, i.e., 3.5 kg and thus β is equal to 3.5 kg. In other words, all defective units observed after the grading operation are considered as residues and β is equal to the total summation of their corresponding weights.

The scenario generation approach described above results in 2^n scenarios for a typical durable product that consists of n different types of components. The probability of each scenario can be calculated as follows.

$$\pi_s = \hat{p}^{\gamma_{ps}+\delta_{ls}}.\hat{q}^{n^p+n^l-\gamma_{ps}-\delta_{ls}}$$

2.3 Two-stage stochastic programming formulation

Herein, the notations are presented. Let Z indicate the set of part suppliers, U indicate the set of raw material suppliers, H indicate the set of module suppliers, I indicate the set of manufacturing centers, J indicate the set of distribution centers, K indicate the set of end-user zones, C indicate the set of collection centers, A indicate the set of disassembly centers, O indicate the set of spare parts markets, M indicate the set of remanufacturing centers, W indicate the set of remanufactured modules markets, B indicate the set of recycling centers, G indicate the set of material recycling centers, E indicate the set of recycled materials markets, D indicate the set of disposal centers, R indicate the set of raw materials in the product.

Let $[Pk_k, Pw_l, Ps_p, Pe_r]$ be unit prices of the brand-new product and recovered components. Let $[fc_c, fa_a, fm_m, fb_b, fg_g, fd_d]$ be fixed costs of opening collection, disassembly, remanufacturing, bulk recycling, material recycling, and disposal centers. Let $[cz_{zp}, cu_{ur}, ch_{hl}, Pr]$ be unit procurement costs of new parts, raw materials, modules from suppliers, and acquisition price of returns. Let $[ci_i, cj_j, cc_c, ca_a, cm_{ml}, cb_b, cg_{gr}, cd_d]$ be unit processing costs at different CLSC facilities. Let $[ti_{zip}, ri_{uir}, si_{hil}, tj_{ij}, tk_{jk}, tc_{kc}, ta_{ca}, ts_{aop}, tz_{aip}, tb_{ab}, tm_{aml}, tg_{agr}, rg_{bgr}, sd_{gd}, rd_{bd}, tx_{mil}, te_{ger}, tw_{mwl}, tu_{gir}]$ be unit transportation costs between network nodes. Let $[dk_k, dw_{wl}, ds_{op}, de_{er}]$ be demands of the brand-new product and recovered components at their corresponding marketplaces. Let $[caz_{zp}, cau_{ur}, cax_{xl}, cai_i, caj_j, cac_c, caa_a, cab_b, cad_d, cam_{ml}, cag_{gr}]$ be capacities of CLSC facilities. Let ϕ_p be the number of part p in each unit of product, μ_r

be the mass of material r in each unit of product, ω_l be the number of module l in each unit of product, ψ be the rate of return, α_r be the mass of recyclable material r in the returned product shipped to material recycling centers from disassembly centers, η_r be the ratio of recyclable material r shipped to material recycling centers from bulk recycling centers, τ_r be the ratio of non-recyclable material r shipped to disposal centers from bulk and material recycling centers, and finally, λ be the mandatory recovery target set by the policymaker.

The first stage location variables in addition to the non-negative physical flows are stated as follows. let Y_c^1 take value one if collection center c is opened and zero otherwise, Y_a^2 take value one if disassembly center a is opened and zero otherwise, Y_m^3 take value one if remanufacturing center m is opened and zero otherwise, Y_b^4 take value one if bulk recycling center b is opened and zero otherwise, Y_g^5 take value one if material recycling center g is opened and zero otherwise, and finally Y_d^6 take value one if disposal center d is opened and zero otherwise. Let X_{ij}^1 denote the quantity of products shipped from manufacturing center i to distribution center j, X_{jk}^2 denote the quantity of products shipped from distribution center j to end-user zone k, X_{kc}^3 denote the quantity of returns shipped from end-user zone k to collection center c, X_{ca}^4 denote the quantity of returns shipped from collection center c to disassembly center a, and X_{agr}^5 denote the quantity of recyclable material r shipped from disassembly center a to material recycling center g.

The second stage decision (recourse actions) variables are as follows. Let QI_{zips} denote the quantity of part p shipped from part supplier z to manufacturing center i in scenario s, NI_{uirs} denote the quantity of material r shipped from material supplier u to manufacturing center i in scenario s, XI_{hils} denote the quantity of module l shipped from module supplier h to manufacturing center i in scenario s, QS_{aops} denote the number of part p shipped from disassembly center a to spare parts market o in scenario s, QZ_{aips} denote the number of part p shipped from disassembly center a to manufacturing center i in scenario s, QM_{amls} denote the quantity of module l shipped from disassembly center a to remanufacturing center m in scenario s, QW_{mwls} denote the number of module l shipped from remanufacturing center m to secondary market w in scenario s, QX_{mils} denote the number of module l shipped from remanufacturing center m to manufacturing center i in scenario s, QB_{abs} denote the quantity of residues shipped from disassembly center a to bulk recycling center b in scenario s, NG_{bgrs} denote the quantity of recyclable material r shipped from bulk recycling center b to material recycling center g in scenario s, QE_{gers} denote the quantity of recycled material r shipped from material recycling center g to recycled material market e in scenario s, QU_{girs} denote the quantity of recycled material r shipped from material recycling center g to manufacturing center i in scenario s, ND_{bds} denote the quantity of residues shipped from bulk recycling center b to disposal center d in scenario s, XD_{gdrs} denote the quantity of material r shipped from material recycling center g to disposal center d in scenario s.

$$\begin{aligned}
\text{Max}\quad & \sum_{j\in J}\sum_{k\in K} Pk_k X^2_{jk} - \sum_{i\in I}\sum_{j\in J}(ci_i + tj_{ij})X^1_{ij} - \sum_{j\in J}\sum_{k\in K}(cj_j + tk_{jk})X^2_{jk} - \sum_{c\in C} fc_c Y^1_c \\
& - \sum_{k\in K}\sum_{c\in C}(cc_c + tc_{kc})X^3_{kc} - \sum_{c\in C}\sum_{a\in A}(ca_a + ta_{ca} + Pr)X^4_{ca} - \sum_{a\in A} fa_a Y^2_a \\
& - \sum_{a\in A}\sum_{g\in G}\sum_{r\in R}(cg_{gr} + tg_{agr})X^5_{agr} - \sum_{m\in M} fm_m Y^3_m - \sum_{b\in B} fb_b Y^4_b - \sum_{g\in G} fg_g Y^5_g \\
& - \sum_{d\in D} fd_d Y^6_d + \sum_{s\in S}\pi_s Q(Y^1_c, Y^2_a, Y^3_m, Y^4_b, Y^5_g, Y^6_d, X^1_{ij}, X^2_{jk}, X^3_{kc}, X^4_{ca}, X^5_{agr}, \boldsymbol{\xi}_s) \qquad (1)
\end{aligned}$$

$$\text{s.t.}\quad \sum_{i\in I} X^1_{ij} = \sum_{k\in K} X^2_{jk} \quad \forall\, j\in J \qquad (2)$$

$$\sum_{j\in J} X^2_{jk} = dk_k \quad \forall\, k\in K \qquad (3)$$

$$\sum_{c\in C} X^3_{kc} = \psi dk_k \quad \forall\, k\in K \qquad (4)$$

$$\sum_{k\in K} X^3_{kc} \geq \sum_{a\in A} X^4_{ca} \quad \forall\, c\in C \qquad (5)$$

$$\sum_{c\in C}\sum_{a\in A} X^4_{ca} \geq \lambda \sum_{k\in K}\sum_{c\in C} X^3_{kc} \qquad (6)$$

$$\sum_{c\in C} \alpha_r X^4_{ca} = \sum_{g\in G} X^5_{agr} \quad \forall\, a\in A, \forall\, r\in R \qquad (7)$$

$$\sum_{j\in J} X^1_{ij} \leq cai_i \quad \forall\, i\in I \qquad (8)$$

$$\sum_{i\in I} X^1_{ij} \leq caj_j \quad \forall\, j\in J \qquad (9)$$

$$\sum_{k\in K} X^3_{kc} \leq cac_c YC_c \quad \forall\, c\in C \qquad (10)$$

$$\sum_{c\in C} X^4_{ca} \leq caa_a YA_a \quad \forall\, a\in A \qquad (11)$$

where $Q(Y^1_c, Y^2_a, Y^3_m, Y^4_b, Y^5_g, Y^6_d, X^1_{ij}, X^2_{jk}, X^3_{kc}, X^4_{ca}, X^5_{agr}, \boldsymbol{\xi}_s)$ is the optimal value of the following problem:

$$\begin{aligned}
\text{Max}\quad & \sum_{a\in A}\sum_{o\in O}\sum_{p\in P} Ps_p QS_{aops} + \sum_{m\in M}\sum_{w\in W}\sum_{l\in L} Pw_l QW_{mwls} + \sum_{g\in G}\sum_{e\in E}\sum_{r\in R} Pe_r QE_{gers} \\
& - \sum_{z\in Z}\sum_{i\in I}\sum_{p\in P}(cz_{zp} + ti_{zip})QI_{zips} - \sum_{u\in U}\sum_{i\in I}\sum_{r\in R}(cu_{ur} + ri_{uir})NI_{uirs} \\
& - \sum_{h\in H}\sum_{i\in I}\sum_{l\in L}(cx_{hl} + si_{hil})XI_{hils} - \sum_{a\in A}\sum_{m\in M}\sum_{l\in L}(cm_{ml} + tm_{aml})QM_{amls} \\
& - \sum_{a\in A}\sum_{b\in B}(cb_b + tb_{ab})QB_{abs} - \sum_{b\in B}\sum_{g\in G}\sum_{r\in R}(cg_{gr} + rg_{bgr})NG_{bgrs} - \sum_{b\in B}\sum_{d\in D}(cd_d + rd_{bd})ND_{bds} \\
& - \sum_{g\in G}\sum_{d\in D}\sum_{r\in R}(cd_d + sd_{gd})XD_{gdrs} - \sum_{a\in A}\sum_{o\in O}\sum_{p\in P} ts_{aop} QS_{aops} - \sum_{m\in M}\sum_{w\in W}\sum_{l\in L} tw_{mwl} QW_{mwls} \\
& - \sum_{g\in G}\sum_{e\in E}\sum_{r\in R} te_{ger} QE_{gers} - \sum_{a\in A}\sum_{i\in I}\sum_{p\in P} tz_{aip} QZ_{aips} - \sum_{m\in M}\sum_{i\in I}\sum_{l\in L} tx_{mil} QX_{mils} \\
& - \sum_{g\in G}\sum_{i\in I}\sum_{r\in R} tu_{gir} QU_{girs} \qquad (12)
\end{aligned}$$

$$\text{s.t.}\quad \sum_{z\in Z} QI_{zips} + \sum_{a\in A} QZ_{aips} = \phi_p \sum_{j\in J} X^1_{ij} \quad \forall\, i\in I, \forall\, p\in P \qquad (13)$$

$$\sum_{u\in U} NI_{uirs} + \sum_{g\in G} QU_{girs} = \mu_r \sum_{j\in J} X^1_{ij} \quad \forall\, i\in I, \forall\, r\in R \qquad (14)$$

$$\sum_{h\in H} XI_{hils} + \sum_{m\in M} QX_{mils} = \omega_l \sum_{j\in J} X^1_{ij} \quad \forall\, i \in I, \forall\, l \in L \tag{15}$$

$$\sum_{c\in C} \gamma_{ps} X^4_{ca} = \sum_{i\in I} QZ_{aips} + \sum_{o\in O} QS_{aops} \quad \forall\, a \in A, \forall\, p \in P \tag{16}$$

$$\sum_{a\in A} QS_{aops} \leq ds_{op} \quad \forall\, o \in O, \forall\, p \in P \tag{17}$$

$$\sum_{c\in C} \beta_s X^4_{ca} = \sum_{b\in B} QB_{abs} \quad \forall\, a \in A \tag{18}$$

$$\sum_{c\in C} \delta_{ls} X^4_{ca} = \sum_{m\in M} QM_{amls} \quad \forall\, a \in A, \forall\, l \in L \tag{19}$$

$$\sum_{a\in A} QM_{amls} = \sum_{w\in W} QW_{mwls} + \sum_{i\in I} QX_{mils} \quad \forall\, m \in M, \forall\, l \in L \tag{20}$$

$$\sum_{m\in M} QW_{mwls} \leq dw_{wl} \quad \forall\, w \in W, \forall\, l \in L \tag{21}$$

$$\sum_{a\in A} \eta_r QB_{abs} = \sum_{g\in G} NG_{bgrs} \quad \forall\, b \in B, \forall\, r \in R \tag{22}$$

$$\sum_{a\in A} QB_{abs} = \sum_{g\in G}\sum_{r\in R} NG_{bgrs} + \sum_{d\in D} ND_{bds} \quad \forall\, b \in B \tag{23}$$

$$\sum_{a\in A} \tau_r X^5_{agr} + \sum_{b\in B} \tau_r NG_{bgrs} = \sum_{d\in D} XD_{gdrs} \quad \forall\, g \in G, \forall\, r \in R \tag{24}$$

$$\sum_{g\in G} QE_{gers} \leq de_{er} \quad \forall\, e \in E, \forall\, r \in R \tag{25}$$

$$\sum_{a\in A} X^5_{agr} + \sum_{b\in B} NG_{bgrs} = \sum_{i\in I} QU_{girs} + \sum_{e\in E} QE_{gers} + \sum_{d\in D} XD_{gdrs} \quad \forall\, g \in G, \forall\, r \in R \tag{26}$$

$$\sum_{i\in I} QI_{zips} \leq caz_{zp} \quad \forall\, z \in Z, \forall\, p \in P \tag{27}$$

$$\sum_{i\in I} NI_{uirs} \leq cau_{ur} \quad \forall\, u \in U, \forall\, r \in R \tag{28}$$

$$\sum_{i\in I} XI_{hils} \leq cax_{hl} \quad \forall\, h \in H, \forall\, l \in L \tag{29}$$

$$\sum_{a\in A} QM_{amls} \leq cam_{ml} Y^3_m \quad \forall\, m \in M, \forall\, l \in L \tag{30}$$

$$\sum_{a\in A} QB_{abs} \leq cab_b Y^4_b \quad \forall\, b \in B \tag{31}$$

$$\sum_{a\in A} X^5_{agr} + \sum_{b\in B} NG_{bgrs} \leq cag_{gr} Y^5_g \quad \forall\, g \in G, \forall\, r \in R \tag{32}$$

$$\sum_{b\in B} ND_{bds} + \sum_{g\in G}\sum_{r\in R} XD_{gdrs} \leq cad_d Y^6_d \quad \forall\, d \in D \tag{33}$$

In model (1)-(33), the objective function is to maximize the expected profit for all realized quality state scenarios. The objective function is composed of the revenue from selling brand-new products and recovered components and recycled materials in addition to the fixed costs of opening facilities as well as processing, procurement, and shipping costs in the CLSC network. Constraint (2)-(3) ensure flow balance at each distribution center and demand satisfaction at each end-user zone. Constraint (4) ensures that all the returned products are collected. Constraint (5) ensures that the total flow to the disassembly facilities, i.e., acquired returns, cannot exceed the total amount of returned products available in collection centers. Constraint (6) ensures that the OEM acquires a substantial portion of the return stream for recovery purposes. This constraint reflects the environmental concerns regarding the harmful effects of leaving used durable products in the environment. Constraint (7) ensures that the total flow outgoing from disassembly centers to all recycling centers is equal to the incoming flow to each disassembly center from all collection centers, multiplied by recyclable mass coefficient α_r. Constraints (8)-(11) are capacity restrictions. Constraints (13)-(15) ensure that the total outgoing flow from each manufacturers is equal to the total incoming flow into this facility from suppliers and reverse channel. Constraints (16)-(19) ensure flow conservation at each disassembly center. Constraints (20)-(21) ensure the flow conservation at each remanufacturers. Constraints (22)-(23) ensure flow conservation at each bulk recycling center. Constraints (24)-(26) are flow conservation restrictions at each material recycling center. Constraints (27)-(33) impose capacity restrictions on supply chain facilities. Constraints (17), (21), and (25) represent partial demand satisfaction of recovered components and recycled raw materials at secondary markets.

3 Numerical results

In this section, we illustrate numerical experiments considering a reverse supply chain associated to the recovery of used washing machines inspired from ([7], [8]). All data sets are available upon request. The washing machine consists of twelve components, namely two modules and ten parts. Thus, the grading process yields 2^{12} or 4096 quality state scenarios which is quite large. We also consider 500 as the size of the reduced set of scenarios after the implementation of the fast forward algorithm. All the algorithms are implemented in C++ using Concert Technology with IBM-ILOG CPLEX 12.60 and all experiments are carried out on an Intel Quad Core 3.40 GHz with 8 GB RAM. We employ five classes of problems, each with 3 randomly generated test instances, for the considered set of scenarios as shown in Table 1. The detailed information on the size of problem classes is shown in Table 2.

Table 1. Problem classes

Class	$\|Z\|$	$\|U\|$	$\|H\|$	$\|I\|$	$\|J\|$	$\|K\|$	$\|C\|$	$\|A\|$	$\|M\|$	$\|B\|$	$\|G\|$	$\|D\|$	$\|O\|$	$\|W\|$	$\|E\|$	$\|S\|$
C1	10	3	2	5	10	60	10	10	10	10	10	5	30	30	30	500
C2	10	3	2	5	10	80	10	10	10	10	10	5	40	40	40	500
C3	10	3	2	5	15	100	15	15	15	15	15	7	50	50	50	500
C4	10	3	2	5	15	120	15	15	15	15	15	7	60	60	60	500
C5	10	3	2	5	20	140	20	20	20	20	20	10	70	70	70	500

3.1 Computational results

An accelerated L-shaped method proposed by the same authors in [6] was implemented to solve the resulting large-scale two-stage stochastic CLSC design model. Further, we also solve all 15 test instances with CPLEX in a maximum time limit of 18000 seconds and within the stopping gap tolerance of 0.5% to

Table 2. Size of the deterministic equivalent problems

Class	Constraints	Continuous Vars.	Binary Vars.
C1	476706	3261650	55
C2	551746	4012050	55
C3	705326	7276475	82
C4	780366	8402075	82
C5	934446	13022300	110

avoid tail-off effect. Table 3 presents computational results. The last column entitled represents the relative difference between the solutions of the accelerated L-shaped algorithm and CPLEX for each test instance.

Table 3. Computational results on problem classes for $|S| = 500$

Class	Accelerated L-shaped			CPLEX		
	Runtime (sec)	Iterations	Profit	Runtime (sec)	Profit	Gap (%)
	320	22	23,583,400	$\geq$ 18000	23,524,200	0.25
C1	170	12	26,655,700	$\geq$ 18000	26,460,800	0.73
	280	19	25,156,500	$\geq$ 18000	24,860,600	1.18
	443	27	35,837,400	$\geq$ 18000	35,460,700	1.05
C2	527	32	34,890,800	$\geq$ 18000	34,703,900	0.54
	224	14	36,404,200	$\geq$ 18000	35,984,800	1.15
	969	32	44,375,400	$\geq$ 18000	4,181,020	90.58
C3	521	17	40,886,900	$\geq$ 18000	1,050,180	97.43
	1119	36	45,205,700	$\geq$ 18000	6,253,750	86.17
	1382	36	51,663,200	$\geq$ 18000	10,866,000	78.97
C4	1154	31	56,215,500	$\geq$ 18000	15,416,000	72.58
	763	21	58,737,500	$\geq$ 18000	19,074,500	67.52
	2835	38	65,911,400	$\geq$ 18000	12,020,400	81.76
C5	2900	41	60,880,800	$\geq$ 18000	9,697,270	84.07
	2348	31	63,561,500	$\geq$ 18000	12,181,600	80.83

As it can been seen in Table 3, the accelerated L-shaped method solves all 15 test instances of different sizes to optimality in a reasonable amount of time while CPLEX is only able to find a feasible solution for some of test problems. In particular, the feasible solutions identified by CPLEX for C3 and C4 are considerably far from the optimal solutions given by the L-shaped method observing the huge gaps in the last column. Note that, in the last class of problems, CPLEX cannot find any feasible solution to the test instances within two hours, denoted by "No solution". This can be explained by the fact that the deterministic equivalent problem which CPLEX attempts to solve involves a large number of recourse problems associated with the representative quality state scenarios. This observation supports the call for an efficient solution approach. As opposed to CPLEX, the proposed accelerated L-shaped algorithm can easily handle realistic size problems such that the average runtime in classes one to four is 656 seconds, verifying the advantage of the refinement strategies.

4 Conclusion

In this paper, we introduced a CLSC network design problem under uncertain quality status of the return stream, which is applicable to the case of durable products, e.g., large household appliances. We proposed a two-stage mixed-integer stochastic program to explicitly incorporate uncertainty. On the methodological side, we first adopted adapted fast forward selection algorithm reduce the number of scenarios in the deterministic equivalent problem and the, we developed a solution method based on L-shaped algorithm, further enhanced with additional acceleration strategies. Our computational experiments demonstrated an outstanding capability of the proposed algorithm.

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