A Game-theoretic Approach for Coalitions Formation in a Collaborative Inventory-Distribution problem

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Abstract. In this paper, we propose a collaborative game for the inventory-distribution problem with multiple distribution centers (DCs) and multiple customer zones. The problem consists in addressing a set of firms that are looking for profitability with a re-configuration for their inventory-distribution network, given a predefined customer base. Possible collaborations between DCs-owners, modeled as a coalition formation problem, can be beneficial in the sense that the sharing of their inventory storage space generates cost minimization regarding the stand-alone situation. Given the substantial savings that can be realized thanks to such collaboration, the main interest of the potential collaborating DCs is to figure out how collaborating group should be formed and how stable alliances/coalitions should be generated. Based on game-theory concepts, the coalition formation problem is set and a solution approach is proposed to, generate core-stable coalitions regarding all partner standpoints. The results obtained underline clearly the impact of this collaboration opportunities on the firms' performance in terms of distribution costs savings as well as revenues increase.

Keywords: Collaboration, Game-theory, Sharing mechanisms, Warehousing problem, Supply networks, Economies of scale

1 Introduction

Inventory management is a critical activity in the supply chain as it acts in linking the material flows between suppliers/producers and customers using a set of deployed platforms. The main goals of the inventory activity are to help firms hedging against uncertainty of their customers' demand and reaching good service level with the surrounding aim to keep minimal the inventory holding costs. Under these circumstances, collaboration between firms on the inventory activity, by sharing storage capacities, is viewed as a beneficial strategy to reach these goals [1]. This benefit can be in the form of improved service level quality to fulfill customers' needs and/or cost saving while pooling inventories [2]. Recently, practical examples shown that if the inventory collaboration is organized, it can positively affect the efficiency of the supply chain. The case of the manufacturer Henkel makes it clear that sharing warehouses can strongly affect the sustainability performance. Henkel uses pooling, which means grouping together flows (order processes, storage, handling, inbound and outbound transportation) from manufactures, which have compatible finished goods. Advantages are increasing the frequency of deliveries and service level as well as less stock on retail warehouse stage. By combining inventories with competitors Henkel reduced retailer stocks by 13%, increased the number of deliveries and reduced carbon dioxide emissions by 300 ton (Henkel, 2009) [11]. Logistics service providers can act as interface between competitors and organize the collaboration. For example, Kuehne-Nagel joined the distribution network of Nestle and Danone, which means that they created a joint warehouse in Toulouse and combined the outbound transportation. A benefit could be registered for all three companies as better usage of warehouse capacity and truck fleet for the logistics service provider, and reduced warehousing and transportation costs for the manufacturers (Nestle, 2010) [12]. In a single firm context, inventory pooling has been extensively studied in the supply chain management literature [6]. It is commonly defined as *the consolidation of multiple inventory locations into a single one. Inventory locations may be associated with different geographical sites, different products, or different customers [5]*. Two types of inventory collaboration could be distinguished: inventory-product pooling [5]

and inventory-capacity pooling [2]. The inventory-product pooling refers to meeting demand of different products with a single, product capable of satisfying all customers needs [9]. The capacity pooling, in the other hand, refers to the practice of pooling demands of different geographic markets into single shared site [5]. Multiple capacity pooling models have been studied in the literature [8] and benefits on firms and customers satisfaction have been widely improved [5]. In such context, Eppen, 1979 [7], studied the inventory pooling/ collaboration problem with one firm and multiple customer markets. The author proposed a model that helps firm owning a pooled or non-pooled operational system. Alptekinoglu et al., 2013, [6] considered pooling strategy in the context of production-inventory systems under a time variability. However, when looking to the literature on the inventory-capacity pooling, we noticed that the problem is mostly studied in the situations of a single firms disposing of multiple storage capacities and has to response various customers demands. One should notice that in this work, this firm makes, individually, the decision about whether or not to pool their customers products [5].
In this paper, we consider the collaboration mechanism based on capacity pooling in an inventory-distribution system where multiple firms are facing a set customers' expecting a high service level, but limited distribution capacities. In order to meet its customers needs, each firm have the option of either operating its own distribution centers (DCs) or forming coalition by sharing partners distribution capacities. Such collaboration is driven by economies of scale resulting from the inventory-holding costs and by revenues increase thanks to higher service levels provided to customers. The inventory-distribution problem considered in this paper can be resumed in terms of the following features:

1. n firms/DCs and m customers zones with demand quantities sensitive to the service level
2. Capacity sharing-based collaboration opportunities with inventory cost savings a cost savings

Due to the existence of multiple decision makers (i.e multiple firms), collaboration is viewed as a coalition formation problem embedded in a cooperative game that helps each firm taking the appropriate decision regarding its inventory positioning and distribution policies. We study two alternative scenarios for the inventory-distribution problem given n firms/DCs and m customer zones: (1) the stand alone scenario: each firm operates its own DCs in response to its customers needs. (2) The collaborative scenario: that consists in forming coalitions, sharing capacities and assigning customers to the most profitable DC pertaining to the coalition. With these scenarios, one can ask the following questions: is collaboration always beneficial to all firms? for each firm, which coalition to join? and if some or all of the firms decide to collaborate and share storage capacity, how to allocate the inventory cost within the shared DCs?. To answer these questions, in this paper we modeled the inventory−distribution problem in its stand−alone and collaborative settings. A collaborative solution approach (CSA)is also developed to solve the coalition formation problem. Several problem instances are addressed to assess the efficiency of collaboration in generating appropriate coalition structures and discuss the questions given above.

The rest of the paper is organized as follows. Section 2 defines the problem and presents the modeling features of the inventory and distributions processes in the stand-alone and the collaborative configurations. Section 3 presents the CSA. Preliminary experiments are stated in Section 4 in order to validate the model and solution approach..

2 Problem description and modelling

The inventory-distribution problem, as shown in Figure 1, can be modeled as a tactical level problem that involves a set of n firms, each selling a distinct products family to m customer zones and, operating a DC from w, the set of available DCs. In other words, the considered inventory-distribution problem refers to the case where each firm must decide how much products to order periodically and where to store them, and then, distribute them to customers zones when needed. In this context, each firm when acting solely has to keep its own inventory level within its located DCs, in order to satisfy its customers' demands. In this paper, we studied the inventory-distribution problem of n firms and m customers zones with a single product family to be ordered from the same supplier base (i.e. assuming that the firms-products are sourced from the same zone and transported with the same technologies and processes). In addition, the paper assumes that, a priori, one DC is operated by each firm in the market area studied, and, without loss of generality, that this

Figure 1: n firms-m customers distribution network

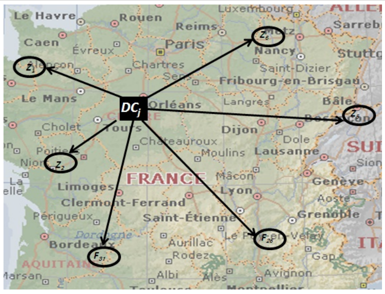

Figure 2: 1 firms-m customers distribution network

DC is owned by the firm. Figure 1 reports the considered distribution network obtained when n firms are acting on the same territory and where each firms operates solely using its DC to satisfy all its customers. In order to maximize its profit during its lifespan, each firm is looking for optimizing for a yearly horizon plan its margin from sales revenues minus total costs. Let N be the set of firms and $i \in N$ denote the index of each firm in N. Let J be the set of DC and $j \in J$ correspond to the index of each DC in this set. Each DC $j \in J$ has a predefined capacity cap_j. Finally, let Z denote the set of customer zones and $z \in Z$ denote the index of each customer zone in Z. For each DC $j \in J$, along the planning horizon, customers in Z order varying quantities of products on a periodic basis. Furthermore, each customer zone is characterized by a demand d_{zj} and a distance δ_{zj} from DC $j \in J$ location.

Demand d_{zj} is sold to customer zone z with a unit price of p_z and it is anticipated in terms of the distance δ_{zj} as given by equation 1. This expression models a sensitivity of the demand level to the distance between the customers zone z and the location of a given DC j.

$$d_{zj} = \begin{cases} d^j(1+\rho_j^1) & \text{if } \delta_0 \leq \delta_{zj} < \delta_1 \\ d^j(1+\rho_j^2) & \text{if } \delta_1 \leq \delta_{zj} < \delta_2 \\ d^j & \text{if } \delta_2 \leq \delta_{zj} < \delta_3 \end{cases} \tag{1}$$

Under the objective of welfare maximization, two scenarios for the inventory management can arise, for each firm/DC:

• *The stand-alone scenario:* which allows satisfying all customers zones from firms own operated DCs. Figure 2 presents such scenario for a given DC $j \in J$ trying to respond to m customers needs from one DC location. Each customer zone is characterized by a demand d_{zj} and a distance δ_{zj} from DC $j \in J$ is location. We denote by D_j the sum of d_z^j orders received by DC $j \in J$. We assume that the cumulative demand D_j received at a given DC from a subset of its assigned customers is characterized by a *Normal process*. As its is the case, the optimal ordering quantity, in each period of the planning horizon, is determined by the so called economic ordering quantity (EOQ) formula. Let Q_j denote the individual ordering quantity computed for the replenishment plan of DC j based on the demand D_j. Hence, a number of orders N_j is determined for each DC $j \in J$. Once the ordering quantity is determined, the DC operator tempts to optimize individually its own revenues and costs for the planning horizon considered by finding the best compromise among ordering, inventory holding and distribution components. Let P_j, R_j and C_j denote the profit, sales revenues, and total tactical costs anticipated for the firm with DC $j \in J$. Cost function C_j is composed of three components: the ordering costs, denoted by C_j^O based on EOQ policy, depends mainly on a fixed ordering cost A, a distance from supplier's location to DC j is location δ_{0j} as well as a unitary distance rate β_2 to ship one unit of product. The transportation costs denoted by C_j^T which includes shipment costs related to anticipated quantity to order, given by $\beta_0 \times Q_j$, and distribution costs depend on traveled distance δ_{zj}, computed a priori by $\beta_1 \times \delta_{zj}$ according to each customer zone. All costs functions are summarized in table 1. Finally, the inventory-holding costs, denoted by C_j^I has the form:

$$C_j^I(Q_j) = h_j \hat{\alpha}_0 Q_j^{\alpha_1} \tag{2}$$

where h_j is the unitary cost of capital to hold one unit of product into DC_j, $\hat{\alpha}_0$ is a positive inventory parameter (could be obtained by regression from historical data) and α_1 (which generally falls in the range $[0.5, 0.8]$) reflects inventory concentration requirements leading to economies of scale [10]. More specifically, higher storage quantities promotes high quality inventory technologies which lead to lower average inventory levels and costs.

•*The collaborative scenario:* in which, firms can share their DCs' storage capacities and maximize their own profits given an economies of scale in their inventory-holding costs. A possible collaboration between two DCs is represented in Figure 3. In such case, collaborating firms can response to their customers' requirements from any DCs pertaining to the coalition. We assume that each customer will be assigned exactly to one DC. The replenishment process from supplier's location to firms' DCs is synchronized through pooling inventory-capacities. As mentioned above, the inventory model is usually designed by the firms/DCs alone but not by anticipating economies of scale that can realized when pooling inventories. Accordingly, in the presence of multiple DCs, the inventory plans can be viewed as a collaborative process in which firms are aiming to identify profitable coalitions to form by anticipating the future revenues and costs. The firms have interest to collaborate in order to increase their revenues when customers demands are increased as given by (1). Accordingly, when firms decide to collaborate, the shared positioning of their DCs provides a significant increase in their proximity to customers zone. More specifically, as given by equation (1) when the demand level is sensitive to the service level, the positioning of inventories impacts on the market, the total ordered demand by customer and consequently, inventory-holding costs are reduced thanks to the economies of scale as summarized in equation (2). This collaborative scenario gives rise to a coalition formation problem that addresses the optimal inventory-distribution plan for each firm by anticipating its future replenishments, inventory-holding policies and distribution operations. Recall that a coalition or alliance characterizes a group of firms that decides to work jointly while sharing their storage capacities and adopting new assignment of the set of customers to DCs. The coalition formation problem aims to answer to two key questions: what is the best coalition to form so that the profit of each collaborating firm is increased? How will collaborative profit be shared among firms in the coalition?

With this in mind, let us now characterize explicitly the coalition formation decision-making problem.

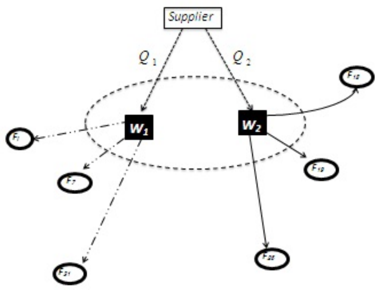

Figure 3. Illustrative solution when two firms are collaborating

Given the demand level and the associated locations of all the customers zones, each DC owner is tempted to evaluate all possible coalitions with regards to partners operating on the market and predisposed to cooperate. The anticipation of a profitable coalition essentially implies sharing among the partners some information about their estimated annual demands and periodic ordered quantities. Each DC owner will operate as a single decision-making unit, to decide and store goods in a shared or the owned DC. Let S denote a given coalition composed by a collection of DCs (i.e collection of firms as we have the assumption of only one owned DC per firm). For a given coalition S, let J_s be the subset of DCs assigned to S and I_s the set of collaborative firms (i.e here we have $I_s = J_s$ as we have exactly one owned DC for each firm). The assigned customer zones to collaborative DCs $j \in J_s$ is designed by Z_j^s and determined according to distance matrix $\delta_{zj}(n \times m)$ as given by equation 3:

$$Z_j^s : j(z) = argmin_{j \in J_s} \delta_{zj}; \forall z \in Z \tag{3}$$

Equation 3 minimizes the traveled distribution distance by affecting each customer to the nearest DC. When a coalition S is formed, the demand and the ordered quantity of each collaborating firm must be adjusted for each DC j pertaining to the coalition. Let D_j^s, Q_j^s and N_j^s denote respectively the joint demand, the quantity ordered and the frequency associated to DC $j \in J_s$. A collaborative profit P_s is associated to S which depends on collaborative revenues R_s, replenishment cost C_s^R, distribution cost C_s^D and inventory-holding cost C_s^H. We denote by $C_{j(s)}^O(D_j^s)$, $C_{j(s)}^T(D_j^s)$ and $C_{j(s)}^I(Q_j^s)$, the ordering transportation and inventory costs associated

to each DC $j \in J_s$, respectively. These functions are detailed in Table 1.
The main objective of the collaborative inventory-distribution problem is to find stable coalitions for each

Notation	*Stand-alone scenario*	*Collaborative scenario*
Demand	$D_j = \sum_{z\in Z} d_{zj}$	$D^s_j = \sum_{z\in Z^s_j} \sum_{j\in J_s} d_{zj}$
Ordered quantity	$Q_j = \sqrt{\frac{2AD_j}{h_j}}$	$Q^s_j = \sqrt{\frac{2AD^s_j}{h_j}}$
Frequency	$N_j = \frac{D_j}{Q_j}$	$N^s_j = N_j$
Profit	$P_j(D_j) = R_j(D_j) - (C^O_j(D_j) - C^T_j(D_j) - N_jC^I_j(Q_j))$	$P_s((D^s_j)) = \sum_{j\in J_s}(R^s_j(D^s_j) - (C^O_{j(s)}(Q^s_j) - C^T_{j(s)}(D_{i(s)}) - C^I_{j(s)}(Q^s_j)))$
Revenues	$R_j(D_j) = \sum_{z=1}^{Z} d^j_z p_z$	$R^s_j(D^s_j) = \sum_{z\in Z^s_j} \sum_{j\in J_s} d_{zj} p_z$
Ordering cost	$C^O_j(Q_j) = \sqrt{2(A+\beta_2\delta^0_j)h_jD_j}$	$C^O_{j(s)}(Q^s_j) = \sqrt{2(A+\beta_2\delta_{0j})h_jD^s_j}$
Transportation cost	$C^T_j(Q_j) = (\beta_0 D_j + \beta_1 \sum_{z\in Z} \delta^j_z)$	$C^T_{i(s)}(D_{i(s)}) = \sum_{j\in J_s} \sum_{z\in Z^s_j}(\beta_0 d_{zi} + \beta_1 \delta_{zj})$
Inventory cost	$C^I_j(Q_j) = h_j\alpha_0 Q_j^{\alpha_1}$	$C^I_{j(s)}(Q^s_j) = h_j\alpha_0 Q^{s\,\alpha_1}_j N^s_j$

Table 1: Problem modelling features

firm. The notion of stable coalition in game theory is based on the seminal concept of the coalition structure core [4], which means that coalition structures are stable in the sense that no firm has an incentive to deviate. Core-stable coalitions are determined by comparing stand-alone profit and shared collaborative profit P^s_j. This later is computed according to the demand of each collaborative firm as follows:

$$P^s_j = \frac{P_s \times D^s_j}{\sum_{j\in J_s} D^s_j} \tag{4}$$

where D^s_j, designs the demand of collaborative DC j in coalition S.
Given the expected profits generated by the two scenarios described above, each firm should decide about its inventory-distribution policy to be either in stand-alone setting or in collaborative setting. In order to help firms finding the most profitable solution, we propose in the next section an efficient cooperative game solution approach which generates stable solutions for the set firms engaged in the coalition formation problem.

3 The collaborative solution approach

We state in what follows, a solution approach for the collaborative inventory-distribution problem.
The coalition formation problem solving is costly, as the number of possible coalitions is $2^n - 1$, when n DCs are involved. After preprocessing phase 1, Step 1 of phase 2 in CSA generates the set of profitable coalitions joined by each DC j. Joining such coalition assumes that the DC's profit P^s_j after cooperating exceeds its profit P_j when acting alone, but it should accept only stable coalitions. Next, Step 2 aims to find

Algorithm 1 The collaborative solution approach (CSA)

1. Input data
Initialize all DCs' data and customers' data

2. Coalition formation problem

- *Stand-alone situation:* Compute for each DC its individual profit based on the customers' assignments
- *Coalitional situation:*
  - $Step_1$: Compare the coalitional and the stand-alone profits after enumerating all possible coalitions
  - $Step_2$:
    - Remove the set of coalitions not respecting the capacity constraint of each DC
    - Determine the set of profitable coalitions E_j for each DC $j \in J$ and regarding all possible coalitions
  - $Step_3$: Determine the set of stable coalitions ϕ by applying the core-stability conditions expressed in equation 6
  - $Step_4$: If ϕ is empty, then the stable coalition is the stand-alone

2. Output data
The best profit for each firm and the coalition to which it pertains

the stable coalitions, that are provided by group of members agree on how to share resulting profits and where no member would prefer to be in another coalition. This step also check capacity constraint at DCs when coalitions are assessed. The core concept seems to be an efficient mechanism to solve such problem by discarding non profitable coalitions. This is done in steps 3 and 4 of the procedure. A coalition will be selected if it is accepted by all DCs-owners of that coalition and discarded otherwise. The core stability of a coalition structure against a deviation from a coalition S is as follows:

$$\sum_{j \in J_s} P_j < P_s \tag{5}$$

$$Cr = \{x = \{S_1, S_2, ..., S_n\}, \forall i \in x; j \in J_s \;\; P_j^s \geq P_j^{s'} \; \forall s' \neq s\} \tag{6}$$

4 Numerical results

In this section, we validate the CSA and examine empirically the solution of the inventory-distribution problem on a set of instances. This latter is implemented in Java language on a Pentium (R) CPU, 2.13 GHz, with 4GB of RAM. The algorithm is run on several problem sizes $w \in [5,..,50]$ and $m \in [30,..,300]$. The other data are randomly generated in the interval $[10,..,20]$ for holding costs, in the interval $[50,..,200]$ for firms' demands, in the interval $[10,..,70]$ for distance between customers zones and DCs' locations and in the interval $[100,..,200]$ for the selling prices. In addition, instances are run with $A = 60$, $\beta_0 = 10$, $\beta_1 = 20$, $\beta_2 = 20$, $\hat{\alpha}_0 = 10$ and $\alpha_1 = 0.59$. Table 2 provides for each pair (w,m) tested, the sum for all firms of their stand-alone profit (denoted *SA* column), the sum for all the coalitions of their joint profit (denoted *Coal* column) and the gap between these two profit. Gap values are reported in the fifth column of Table 2 and computed using the following formula:

$$Gap = \sum_{s \in Cr} \frac{P_s - \sum_{j \in J_s} P_j^s}{\sum_{j \in J_s} P_j^s} \times 100 \tag{7}$$

The results generated by our algorithm are very promising for the following reasons:

• *Gap quality:* Computational results show that the collaborative profit is always greater than stand-alone profit, as presented in Figure 4 and that gaps between stand-alone and collaborative profit are positive ranged in [16.79%,. . . , 52.69%], which implies the profitability of cooperation in all cases studied. This

		Profit			$\lvert Cr \rvert$	$\lvert LC \rvert$
w	m	*SA*	*Coal*	$Gap(\%)$		
5	30	322456.34	454236.77	40.86	3	2
10	60	423376.67	534269.54	26.19	5	3
15	90	476234.27	702156.56	47.43	10	4
20	120	562756.32	693174.20	23.17	9	5
25	150	934175.2	1426483.11	52.96	11	5
30	180	1638824.28	2035521.37	24.20	13	6
35	210	2934610.71	3427448.04	16.79	15	9
40	240	3526881.92	4732540.76	34.18	18	9
45	270	3723396.21	5003257.88	34.37	21	11
50	300	4024833.27	6027712.23	49.76	22	11

Table 2. Stand-alone v collaborative profit

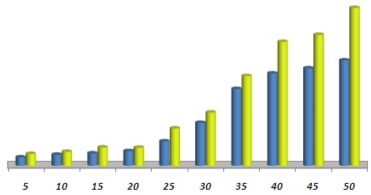

Figure4. Gap values for SA and collaborative scenarios

profitability is mainly realized by the economies of scale in the inventory costs and the revenues increase. Figure 4 further illustrate the gap difference between the SA and the collaborative scenarios.

• *Degree of collaboration:* For each instance, the algorithm always generates a stable coalition structure. The two last columns of Table 2 highlight, respectively, the number of coalitions and the size of the largest one observed in the generated core-stable structure. For instance, when $w = 5$, the algorithm generates a core structure composed of three coalitions where the largest coalition groups two DCs. In fact, we note from table 2 that:
− The size of the core-stable structure is ranging between [3,. . . , 22]. Evidently, we can see that $|Cr|$ increases with the problem size which enhances the collaboration efficiency in the market with multiple DCs. Besides, we can induce that in average 46.18% of DCs find that cooperating is profitable.
− The size of the largest coalitions, $|LC|$, is ranging in the interval [2,. . . , 11] which means that the grand coalition is not reached in these tests. This result can be explained by the limited DCs' capacities. For a

		Traveled distance	
n	m	*SA*	*Coal*
5	10	2102.4	1247.04
10	20	2953.5	2242.35
15	30	2953.5	2307.69
20	40	2953.5	2390.85
25	50	6728.4	4924.8
30	60	6728.4	4879.42
35	70	6728.4	5022
40	80	10282.3	8424.9
45	90	10282.3	8624.7
50	100	13363.2	10837.8

Table 3. Stand-alone vs collaborative distance

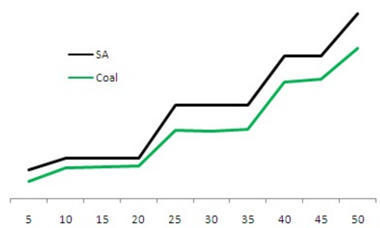

Figure 5. A comparison of stand-alone vs collaborative traveled distance

better understanding of the impact of the inventory pooling on the customer service level, we present in Table 3, for each pair (w,m), the sum of the traveled distances (in kilometer) to distribute the product from the assigned DCs to all the customer zones. The third column provided the traveled distance in the stand-alone scenario whereas column four provides the value in the cooperative scenario. Figure 5 illustrates the efficiency of collaborative scenario in comparison to the stand-alone one in reducing the total shipments distance and thus, increasing the customer service level. Finally, based on the values reported in Table 3, we note that forming coalition reduce in average the traveled distance by 21.79%.

5 Conclusion

The inventory-distribution problem studied in this paper considers the positioning of firms inventories at owned DCs or those of collaborating firms in order to service more profitably a number of customers zones. Collaboration is viewed as a potential concept that aims to pool multiple storage capacities and benefit from service level increase and inventory and distribution costs reduction. We proposed in this paper a game theoretical approach to solve the coalition formation problem when several firms are involved. Several problem sizes were tested with the proposed algorithm in order to quantify its performance in producing stable coalitions. Experimental results showed that the proposed algorithm converges to stable coalition structures in all the cases, and proved that such solutions belong to the core of the cooperative game. Promising results were obtained regarding the increase of profitability for firms in collaborative settings. Also in terms of service level performance, a significant diminution of the proximity between DCs and customers locations was observed under coalitional setting . However, our calibration shown that when the problem size increases, the exact algorithm cannot generate stable coalitions in a reasonable time. In such cases, a heuristic method must be developed in the future to consider larger problem sizes. In addition, alternative sharing mechanisms would be explored in the near future and their impact on the firms' coalitional behavior be tested.

References

[1] Guajardo M., Ronnqvist M., Halvorsen A.M, and Kallevik. S.I: Inventory management of spare parts in an energy company. Journal of Operation Research Society, 66(2):331–341, (2015)

[2] Wang W. and Yue S.: An inventory pooling model for spare units of critical systems that serve multi-companies. Transportation Research Part E: Logistics and Transportation Review, 76:34–44,(2015)

[3] Krichen S., Laabidi A., and Ben Abdelaziz F: Single supplier multiple cooperative retailers inventory model with quantity discount and permissible delay in payments. Computers and Industrial Engineering, 60(1):164–172, (2011)

[4] R. J. Aumann And J. H. Dreze. 'Cooperative games with coalition structures',International Journal of Game Theory, 3 , 217–237(1974)

[5] S. Benjaafar, W. L. Cooper and J. S. Kim, "On the Benefits of Pooling in Production-Inventory Systems," Management Science, 51, 548–565, (2005)

[6] A. Alptekinoglu, A. Banerjee, A. Paul, N. Jain, "Inventory Pooling to Deliver Differentiated Service" Manufacturing and Service Operations management , 15, 33–44 (2013).

[7] G. D. Eppen, "Effects of Centralization on Expected Costs in a Multi-Location Newsboy Problem. Management Science", 25(5), 498–501 (1979).

[8] Q., Liu, and G. Van Ryzin. "Strategic Capacity Rationing to Induce Early Purchases". Management Science., 54(6), 1115–1131 (2008).

[9] J. A. Alfaro and C. J. Corbett. "The Value of SKU Rationalization in Practice (The pooling effect under suboptimal inventory policies and nonnormal demand)". Production and Operations Management, 12(1), 12–29 (2003).

[10] J. F. Shapiro and S. N. Wagner. "Strategic inventory optimization". Journal of Business Logistics, 30(2), 161–173 (2009).

[11] Henkel. (2010). Pooling: A successful collaborative approach. In Presentations from the 2010 European Horizontal Collaboration in the Supply Chain Summit. Available at http://events.eft.com/oldsites/SCHC/2010/presentations2010/day1/1.4-Nico-Eyers-Henkel.pdf [Accessed March 15, 2013].

[12] Logistics Manager. (2010). Nestle and Mars join forces in logistics collaboration. Available at http://www.logisticsmanager.com/Articles/12866/Nestl+in+logistics+collaboration.html [Accessed March 15, 2013].